Test 2 Typed lambda calculi 2006 **November 15 2006**

1. Let $\mathcal{D} =< \mathcal{D}, \sqsubseteq>$ be an ω -algebraic lattice.
Let $\mathcal{K}(\mathcal{D}) = \{a \in \mathcal{D} \mid a \text{ compact}\}$. Define the relation $\leq$ on $\mathcal{K}(\mathcal{D})$ by

$$a \leq b \quad \Leftrightarrow \quad b \sqsubseteq a.$$

(i) Show that $(\mathcal{K}(\mathcal{D}), \leq)$ is a partial order such that for two elements $a, b \in \mathcal{K}(\mathcal{D})$ the glb $a \cap b$ exists.
(ii) Show that there is a greatest element $\top$ in $(\mathcal{K}(\mathcal{D}), \leq)$.

2. Give a direct derivation of the statement

$$\vdash^{BCD}_{\cap^\top} ((\lambda x.xxx)\mathsf{I}) : A {\rightarrow} A$$

where $A \in \mathbb{T}^{BCD}_{\cap}$. You may not use ($\beta$-exp).

3. Show that the rule $({\rightarrow}L)$ is admissible in $\lambda^{BCD}_{\cap^\top}$, that is, give a proof of
If $\quad \Gamma, y : C \vdash P : D \ \& \ \Gamma \vdash N : E \ \& \ z \notin \mathrm{Dom}(\Gamma)$
then $\quad \Gamma, z : E{\rightarrow}C \vdash P[y := zN] : D.$