17. Remember the definition of $\ulcorner M \urcorner$ by Mogensen

$$\begin{array}{rcl} \ulcorner x \urcorner & = & \lambda e.eU^3_1 xe \\ \ulcorner MN \urcorner & = & \lambda e.eU^3_2 \ulcorner M \urcorner \ulcorner N \urcorner e \\ \ulcorner \lambda x.M \urcorner & = & \lambda e.eU^3_3 (\lambda x.\ulcorner M \urcorner)e. \end{array}$$

(i) Verify that $\ulcorner M \urcorner$ is in normal form.
(ii) Prove: $M \not\equiv N \Rightarrow \ulcorner M \urcorner \neq \ulcorner N \urcorner$.
(iii) Investigate whether $(\lambda x.\ulcorner x \urcorner)M = \ulcorner M \urcorner$.

18. Construct λ-terms M_1, M_2, M_3 and M_4 such that

$$\begin{array}{rcl} M_1 & = & M_1 \quad M_1 \\ M_2 & = & \ulcorner M_2 \urcorner \; M_2 \\ M_3 & = & M_3 \; \ulcorner M_3 \urcorner \\ M_4 & = & \ulcorner M_4 \urcorner \ulcorner M_4 \urcorner. \end{array}$$

19. Proof the following lemma that is used in Lambda Calculi with Types, Section 2, in the proof of the Church Rosser Theorem for untyped lambda calculus.
Let $M, M', N, L \in \underline{\Lambda}$. Then
(i) Suppose $x \not\equiv y$ and $x \notin FV(L)$. Then

$$M[x := N][y := L] \equiv M[y := L][x := N[y := L]].$$

(ii)

$$\varphi(M[x := N]) \equiv \varphi(M)[x := \varphi(N)].$$

(iii)

$$M \twoheadrightarrow_{\underline{\beta}} N \Rightarrow \varphi(M) \twoheadrightarrow_{\beta} \varphi(N).$$

20. Prove the Generation Lemma for $\lambda\!\rightarrow$.
(i) $\Gamma \vdash x : A \Rightarrow (x : A) \in \Gamma$.
(ii) $\Gamma \vdash MN : B \Rightarrow \exists A[\Gamma \vdash M : (A{\rightarrow}B) \;\&\; \Gamma \vdash N : A]$.
(iii) $\Gamma \vdash \lambda x.M : C \Rightarrow \exists A, B[\Gamma, x : A \vdash M : B \;\&\; C \equiv (A{\rightarrow}B)]$.