

10. Show that there is no term P_2 such that $P_2(xy) =_{\lambda} y$.

11. Show that given terms A_1, A_2, A_3 there exists a term H such that the scheme of primitive recursion of footnote 5.10 in Reflection and its Use is valid.

$$\begin{aligned} H(F_L x) &= A_1 x(Hx) \\ H(F_p xy) &= A_2 xy(Hx)(Hy) \\ H(F_{!} x) &= A_3 x(Hx) \end{aligned}$$

12. Construct terms P_1 and P_2 such that for all terms M, N

$$P_1 \sqsupseteq MN^{\perp} = M \ \& \ P_2 \sqsupseteq MN^{\perp\perp} = N.$$

13. Show that there exists a term **Num** such that for all terms M
 $\mathbf{Num} \sqsupseteq M^{\perp} = \sqsupseteq\sqsupseteq M^{\perp\perp}$.

14. Prove $\forall M \exists N [N \text{ in } \beta\text{-nf and } N \mathbf{I} \rightarrow\rightarrow_{\beta} M]$.

15. Let $A \equiv \lambda axz.z(aax)$ and $M \equiv AAx$. Pove that M does not have a normal form.

16. $M \in \Lambda^{\tau}$ is in long $\beta\eta$ -normal form iff

$$M \equiv \lambda x_1 \dots x_n. x M_1 \dots M_m,$$

where $x M_1 \dots M_m$ has type 0 and each M_i is in long $\beta\eta$ -normal form.

FAKT Each $M \in \Lambda^{\tau}$ is $\beta\eta$ -equal to a (unique) long $\beta\eta$ -normal form.

Let x be a variable of type $(0 \rightarrow 0) \rightarrow ((0 \rightarrow 0) \rightarrow 0) \rightarrow 0$. Find the long $\beta\eta$ -normal form of x .