Exercises Typed lambda calculi 2006 **September 20 2006**

1. Verify that in the reduction of $(\lambda x.xx)\mathbf{c}_1$ to normal form one needs alpha conversion.

2. Let $\mathbf{W} \equiv \lambda xy.xyy$. Construct the reduction graph for $\mathbf{WWW}$.

3. Construct types for $\mathbf{S}$, $\mathbf{K}$, $\mathbf{SK}$.

4. Prove
   (i) $(\mathbf{c}_n x)^m(y) = x^{n*m}(y)$
   (ii) $(\mathbf{c}_n)^m(x) = \mathbf{c}_{(n^m)}(x)$ for $m > 0$
   (iii) $\mathbf{A}_+\mathbf{c}_n\mathbf{c}_m = \mathbf{c}_{n+m}$
   (iv) $\mathbf{A}_*\mathbf{c}_n\mathbf{c}_m = \mathbf{c}_{n*m}$
   (v) $\mathbf{A}_{exp}\mathbf{c}_n\mathbf{c}_m = \mathbf{c}_{n^m}$ for $m > 0$

5. Find a term of length about 40 cm whose normal form has length more than 10^{10} light years. Each symbol takes 0.3 cm and the speed of light is 300000 km/sec. [Hint use the previous exercise]