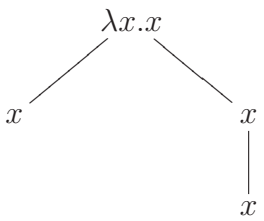
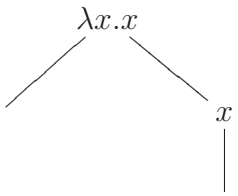
Lambda Calculus, Week 4

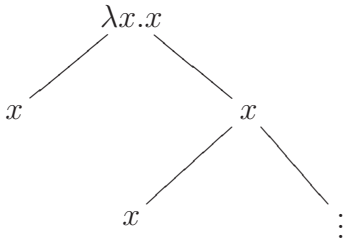
In-class problems

1. Draw the Böhm-trees of the following terms.
   (a) KISS.
   (b) SSS.
   (c) $H\mathbf{c}_0$ with $Hnx \twoheadrightarrow_\beta \langle x, H(\mathsf{S}^+n)(zx)\rangle$. Write down such an H.

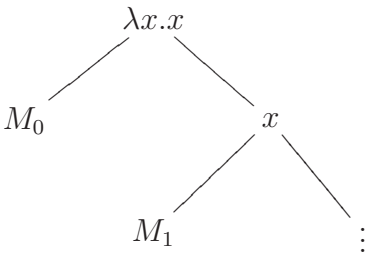
2. Find terms with the following Böhm-trees.

(a)


(b)


(c)


3. Show that there is a term P having the following BT.



, with $M_n = \texttt{true}$, if n is a prime,
$= \text{false}$, else.

Take-home problems

1. Construct a term $M \in \Lambda^{\emptyset}$ such that its $BT(Mx)$ is the binary tree with at each node an x.

2. Goldbach's conjecture is the statement G

   Every even number $n > 3$ is the sum of two primes

   (a) Show that there is a term M such that $\mathrm{BT}(M)$ is finite iff G.

   (b) Show that there is a term N such that $\mathrm{BT}(N)$ is finite iff $\neg G$.

3. Show that $\boldsymbol{\lambda}$ with any of the following axioms becomes inconsistent, i.e. one can derive any equation.

   (a) $\mathsf{I} = \mathsf{K}$.

   (b) $\mathsf{I} = \mathsf{S}$.

   (c) $\mathsf{K} = \mathsf{S}$.

   (d) $\Delta = \mathsf{K}$, with $\Delta := \lambda x.xx$.

   (e)* [Jacopini] $\mathsf{I} = (\lambda x.xxx)(\lambda x.xxx)$.
   [By contrast adding the axiom $\mathsf{I} = \Delta\Delta$ is consistent.]