

Addenda for the sixth imprinting

The variable convention (keeping the names of bound variables as much different as convenient from those of the free variables) is used throughout the book and is convenient in an informal precise setting. This method was brought to my attention by Thomas Ottmann in 1972 and ever since I used it without referring to him (as it is so natural). After the appearance of this book the convention became baptized with my name. This is unjustified to Ottmann and for this reason I mention explicitly his name in these corrections.

Each row in the list below has generally five items: the page number; the line number skipping to count the lines of pictures (a negative number indicates that one has to count from below); the item to be changed; the sign ‘ \mapsto ’; the modified item. In some cases that are self-explanatory a different way of indicating the erratum was used.

A wealth of corrections came from Harold Hodes and participants of a seminar directed by Hidetaka Kondoh: Hironobu Kuruma, Jun-ichi Matsuda, Yuuji Nagamatsu, Takanori Nishio and Tetsuo Tanaka. Other corrections were found by Ingemar Bethke, Pierre-Louis Curien, Herman Geuvers, Bart Jacobs, Gerard Renardel, Piet Rodenburg and Yiqing Zhu. The more important ones are indicated by the symbol \blacktriangleright in the margin.

A few words about progress in theory will be given. In section 6.5 the double fixed-point is stated and proved in two different ways. The first proof is a proof also valid in the λ -calculus. The second proof easily generalizes to the n -fold case. Here we present a third proof, due to Smullyan, that has both virtues.

THEOREM (Multiple fixed-point theorem). Given $F_1, \dots, F_n \in \Lambda$. Then there are $A_1, \dots, A_n \in \Lambda$ such that

$$\begin{aligned} A_1 &= F_1 A_1 \dots A_n \\ &\dots \\ A_n &= F_n A_1 \dots A_n \end{aligned}$$

PROOF. Given \vec{F} , define by the ordinary fixed-point theorem a term A such that

$$A = \lambda f \vec{a}. f(A a_1 \vec{a}) \dots (A a_n \vec{a}),$$

where $\vec{a} = a_1, \dots, a_n$. Take $A_i \equiv A F_i \vec{F}$. Then indeed for $1 \leq i \leq n$ one has

$$\begin{aligned} A_i &\equiv A F_i \vec{F} \\ &= F_i (A F_1 \vec{F}) \dots (A F_n \vec{F}) \\ &\equiv F_i A_1 \dots A_n. \blacksquare \end{aligned}$$

A second result is the solution of several hundred pages in the thesis of Enno Volkerts to problem 21.4.9, see Folkerts [1998].

THEOREM. Let F be a closed term considered as a map $\Lambda^\circ / =_{\beta\eta} \rightarrow \Lambda^\circ / =_{\beta\eta}$. Then

$$F \text{ is a bijection} \iff F \text{ is } \beta\eta\text{-invertible.}$$

Finally conjecture 17.4.15, concerning the place in the projective hierarchy of the λ -theory $\mathcal{H}\omega$ axiomatized by equating all unsolvables and the ω -rule, is proved by a complex argument due to Intrigila and Statman [2004].

THEOREM. $\mathcal{H}\omega$ is a Π_1^1 -complete λ -theory.

This settles most open problems of the book. One conjecture that remains open is the range property for \mathcal{H} , i.e. the question whether for a closed term F its range modulo equating the unsolvables has cardinality either 1 or \aleph_0 .

Nijmegen
Henk Barendregt

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Errata

Preface & Contents		
x	16	ω -rule $\lambda\eta$ \mapsto ω -rule in $\lambda\eta$
xiv	In diagram	Add solid line from 10 to 14.
Chapter 1		
3	-13	18.5.30 \mapsto 18.4.30
4	-21	Aczel [1980]. \mapsto Aczel [1980]. See also Barendregt et al. [1993] for progress on Curry's program.
	-19	[1980] \mapsto [1979]
10	-11	$D = (D', \sqsubseteq')$ \mapsto $D' = (D', \sqsubseteq')$
13	3	(x, x'_0) \mapsto $\langle x, x'_0 \rangle$
15	4	is the least fixed point of f . \mapsto <i>is the least fixed point of f.</i>
	-4	cpo's. \mapsto cpo's if $f_i(\perp) = \perp$ (strictness).
16	-11	<i>retract</i> of D \mapsto <i>retract</i> of D , notation $X \triangleleft D$,
18	1.2.28	<i>coherent</i> \mapsto <i>coherent</i> (or <i>consistently complete</i>)
	-2	$x \ll y$ \mapsto $x \ll y^1$
21	1.3.15	coherent algebraic cpo. \mapsto coherent ² algebraic cpo.
►	21	1.3.16(i) This is incorrect, but holds if each bounded set $Y \subseteq X$ (i.e. $\exists x \in X. Y \sqsubseteq x$) has a supremum in X ; Jung [1989]
Chapter 2		
24	8	$F(WW) = FX$ \mapsto $F(WW) \equiv FX$
►	30	5 and the \mapsto and, if $n > 1$, then the
►	26	-9 VARIABLE CONVENTION \mapsto OTTMANN VARIABLE CONVENTION
35	2	λ \mapsto $\boldsymbol{\lambda}$
36	17	<i>Par abus de langage</i> \mapsto <i>Par abus de langage</i>
41	15	I \mapsto \mathbf{l}
46	7	<i>Applications of CL to λ</i> \mapsto <i>Bases and enumeration</i>
	-4	$\psi(n)$ \mapsto $\lceil \psi(n) \rceil$
47	7	M \mapsto M
48	-11	Böhm out technique \mapsto The Böhm out technique
48	-2	for the \mapsto in the

¹This definition is due to Scott [1972]. If D is a continuous lattice, then it is equivalent to

$$x \ll y \Leftrightarrow \forall \text{ directed } X \subseteq D. [y \sqsubseteq \bigsqcup X \Rightarrow \exists z \in X. x \sqsubseteq z],$$

see Gierz et al. [1980], p. 110-111.

²The condition of coherence may be dropped.

Chapter 3		
51	2	$R \mapsto \succ$
57	3.1.22(ii)	$R\text{-}\infty(M) \mapsto R\text{-}\infty(M) \text{ or } \infty_R(M)$
	3.1.22(iii)	$R \mapsto \mathbf{R}$
67	2	corollary \mapsto theorem
	10	Conservation theorem \mapsto The conservation theorem
71	-3	Sequentiality \mapsto Sequentiality and stability
72	-4	$\beta\eta\Omega$ -reduction $\mapsto \beta\eta\Omega$ -reduction
73	1	Δ -reduction \mapsto Delta reduction
75	19	$\exists x_1, \dots, x_n x = x_1 \succ x_2 \mapsto \exists x_0, \dots, x_n x = x_0 \succ x_1$
Chapter 4		
83	14	17.2 \mapsto 16.2
	-10	16. \mapsto 16.1.
	-1	$\simeq_\eta \mapsto \widetilde{\simeq}_\eta$
84	-4	ω -rule $\mapsto \omega$ -rule in $\lambda\eta$
85	1	Omega incompleteness \mapsto The ω -rule and $\mathcal{H}\eta$
	-2 (2 \times)	$I \mapsto \mathbf{I}$
Chapter 5		
86	11	Plotkins \mapsto Plotkin's
87	17	Scott [1980] \mapsto Scott [1980a]
90	14	$(\lambda^*x, Q) \mapsto (\lambda^*x.Q), \text{ otherwise.}$
91	6	$\mathfrak{M}_1 \mapsto$ (iv) \mathfrak{M}_1
	8	(iv) \mapsto (v)
	5.1.14 all over	} 5 \times $\llbracket \cdot, \cdot \rrbracket \mapsto (\cdot, \cdot)$ (respectively)
92	-5	
	-1	$\llbracket \cdot \rrbracket^{\mathfrak{M}} \mapsto \llbracket \cdot \rrbracket^{\mathfrak{M}}$
93	(6 \times)	$\lambda \vdash \mapsto \lambda \vdash$
	-4	$\lambda \vdash A_\lambda = B_\lambda \Rightarrow \mathfrak{M} \models A_{\lambda,CL} = B_{\lambda,CL}, \text{ by 1,} \mapsto$ $\lambda \vdash A_\lambda = B_\lambda \Rightarrow \mathfrak{M} \models A_\lambda = B_\lambda, \text{ by 1,}$ $\Rightarrow \mathfrak{M} \models A_{\lambda,CL} = B_{\lambda,CL}, \text{ by definition of } \llbracket A_\lambda \rrbracket_\rho^{\mathfrak{M}}$
	-1	$\Lambda(\mathfrak{M}) \mapsto \Lambda(\mathfrak{M}_1)$
94	-4	5.5.8 \mapsto 5.6.8
	-1	$\langle \text{not exactly} \rangle \mathcal{M} \models \mapsto \mathfrak{M} \models$
	-1 (2 \times)	$\lambda \mapsto \lambda^*$
95	1	Meyer [1980] \mapsto Meyer [1982]
	2	Scott [1980] \mapsto Scott [1980a]
	5.2.8 (11 \times)	$\lambda \mapsto \lambda^*$
	5.2.9 (8 \times)	$\lambda \mapsto \lambda^*$
	5.2.10 (4 \times)	$\lambda \mapsto \lambda^*$

Chapter 5 (continued)

96	all over ($3\times$)	\mathbf{K}, \mathbf{S}	\mapsto \mathbf{K}, \mathbf{S} , (respectively)
	$-11, -10, -9$	λ	\mapsto $\boldsymbol{\lambda}$
97	1	$\llbracket \cdot \rrbracket$	\mapsto (\cdot) (respectively)
	-5	Jacopini [1975a]	\mapsto Jacopini [1975]
	$-4, -3, -2$	\mathbf{K}	\mapsto \mathbf{K}
	$-4, -3, -2, -1$	\mathbf{S}	\mapsto \mathbf{S}
98	4	\mathbf{K}	\mapsto \mathbf{K}
		\mathbf{S}	\mapsto \mathbf{S}
99	8	$\mathbf{S}yxz$	\mapsto $\mathbf{S}xyz$
	13	$(\lambda)_c \vdash M = N \Leftrightarrow \lambda \vdash M = N$	\mapsto $(\boldsymbol{\lambda})_c \vdash A = B \Leftrightarrow \boldsymbol{\lambda} \vdash A_\lambda = B_\lambda$
	-1	\mathbf{R} -axiom.	\mapsto \mathbf{R} -ax.
100	11	$\in \lambda$	\mapsto $\in \Lambda$
	15	ξ -ax	\mapsto $\boldsymbol{\xi}$ -ax
	-2	ξ -ax	\mapsto $\boldsymbol{\xi}$ -ax
	$-2, -1$	-rule	\mapsto -rule
101	2	By corollary 5.2.23.	\mapsto Left to the reader.
	$3 - 6$ ($4\times$)	ω	\mapsto $\boldsymbol{\omega}$
	6	theorem 4.1.15(i).	\mapsto proposition 4.1.15(i).
	8	-rule	\mapsto -rule
	10	$\mathcal{T} \models \mathbf{R}$	\mapsto $\mathcal{T} \vdash \mathbf{R}$
	11	5.2.12(ii)	\mapsto 5.2.12(iii)
103	-11	$(x := \llbracket N \rrbracket_\rho)$	\mapsto $(x := \llbracket z \rrbracket_{\rho(z := \llbracket N \rrbracket_\rho)})$
104	4	$\Lambda(\mathfrak{M})$	\mapsto $\Lambda(\mathfrak{M}_1)$
105	5	$x.y$	\mapsto $x \cdot y$
	14	$\llbracket D \rrbracket$	\mapsto $\llbracket P \rrbracket$
107	-13	5.7.7.	\mapsto 5.8.7.
		18.5.29	\mapsto 18.4.29
		18.4.31	\mapsto 20.6.22
	-7	categorical	\mapsto categorical
108	5	p_2)	\mapsto p_2)
	7	$A, B \in C$	\mapsto $A, B \in C$
	14	\langle Delete \rangle It then follows that the same holds for all $f, g : A \rightarrow B$.	
109	-11	λ	\mapsto $\boldsymbol{\lambda}$
110	8	by 4.1.(1)	\mapsto by the note after 5.5.1.(ii)
111	8, 9	\langle Delete two lines. \rangle	
	10	$\Rightarrow \llbracket P \rrbracket_{\Delta, x} = \llbracket Q \rrbracket_{\Delta, x}$	\mapsto $\llbracket P \rrbracket_{\Delta, x} = \llbracket Q \rrbracket_{\Delta, x}$, by the IH,
	13	$\mathfrak{M}(C)$	\mapsto $\mathfrak{M}(C)$
	$-5, -4$ ($2\times$)	\Rightarrow	\mapsto $\Rightarrow \forall x \in \mathfrak{M}$

Chapter 5 (continued)			
112	6	$F \circ G = \text{id},$	$\mapsto F \circ G = \text{id}, G$ is mono,
	8	g	$\mapsto g,$ since p_2 is epi.
113	5.5.10	M	$\mapsto P$
		N	$\mapsto Q$
114	-16	$\mathbf{1}$	$\mapsto \mathbf{1}$
	-8	Scott [1980]	\mapsto Scott [1980a]
115	5.5.7 until end of 5.6, except 5.6.3	$\mathbf{I, K, S}$	\mapsto I, K, S (respectively)
	-5	$t \rightarrow \mathbf{I}$	$\mapsto T \rightarrow \mathbf{I}$
117	9	Scott [1980]	\mapsto Scott [1980a]
	-10	$\mathbf{1}$	$\mapsto \mathbf{1}$
	5.6.3	λ	$\mapsto \lambda^*$
118	5.6.3	λ	$\mapsto \lambda^*$
	14	$\mathbf{1}$	$\mapsto \mathbf{1}$
	16	$\mathbf{K1}_n$	$\mapsto \mathbf{K1}_p$
	17	$\mathbf{1}_n$	$\mapsto \mathbf{1}_p$
	-4	λ	$\mapsto \lambda^*$
120	13	$\mathbf{1}$	$\mapsto \mathbf{1}$
124	14	(X, \cdot, λ)	$\mapsto (X, \cdot, \sqsubseteq)$
	-1	\mathbf{B}	$\mapsto \mathcal{B}$
125	-12	is	\mapsto vs.
126	5	$M(\mathcal{T})$	$\mapsto \mathfrak{M}(\mathcal{T})$
	18	\mathcal{B}	$\mapsto \mathfrak{B}$
127	18	21.4 Exercises	\mapsto 21.4 Exercises
	-7	$\mathcal{F} = (X, \cdot)$	$\mapsto \mathfrak{M} = (X, \cdot)$
128	15, 16	\subsetneq	$\mapsto \sqsubsetneq$
	21	$\mathbf{I, K, S, \Omega}$	\mapsto I, K, S, Ω
Chapter 6			
150	-16	2.3.5	\mapsto 2.4.5

Chapter 7			
152	18	$CL \vdash$	$\mapsto \mathbf{CL} \vdash$
	–8	$(\lambda^*x.Q).$	$\mapsto (\lambda^*x.Q),$ otherwise.
153	–8	Suppose $\vec{x} \notin \text{FV}(\vec{Q})$. Then	\mapsto Then
155	2	remark 3.1.7	\mapsto definition 3.1.5
	8	λ -term	\mapsto λ -term
	§7.2	M, N, L	$\mapsto P, Q, R$ (respectively)
156	–1	$\lambda \vdash$	$\mapsto \lambda \vdash$
157	2	$\lambda \vdash$	$\mapsto \lambda \vdash$
161	11	$= \mathbf{S(KK)(S(S(KS)(S(KK)(SKK)))(K(SK K)))} \mapsto$ $\mathbf{S(KK)(S(S(KS)K)(K(SK K)))}$	
▶	–1	$\lambda x.l \mapsto_{\beta} \lambda x.l$	$\mapsto \lambda x.lx \mapsto_{\beta} l$
▶	162	$\mathbf{S(KI)(KI)} \not\mapsto_w \mathbf{KI}$	$\mapsto \mathbf{S(KI)I} \not\mapsto_w \mathbf{I}$
162	8	w-solvable	$\mapsto w$ -solvable
	–5	$S's$	$\mapsto \mathbf{S}'s$
163	4	I	$\mapsto \mathbf{I}$
	16	K, S	$\mapsto \mathbf{K, S}$ (respectively)
	–7	$\xrightarrow{!}$	$\mapsto \xrightarrow{!}$
	–5	K, S	$\mapsto \mathbf{K, S}$ (respectively)
Chapter 8			
162	–2	Pettorossi	\mapsto Pettorossi
167 – 168	(6×)	#	$\mapsto \#$
169	–9	PROOF.	\mapsto PROOF. (i)
171	13	$x \notin F$	$\mapsto x \notin \text{FV}(F)$
173	9	$N_1N_2 \dots N_n$	$\mapsto N_1N_2 \dots N_k$
177	2	\vec{x}, \vec{w}	$\mapsto \vec{x}\vec{w}$
178	7	$M_{CL}N_{CL}$	$\mapsto M_{CL}\vec{N}_{CL}$
179	13	one has ψ	\mapsto one has ϕ
180	11	occurrences if redexes	\mapsto occurrences of redexes
181	2	$M = M_0$	$\mapsto M \equiv M_0$
	10	$n \in \mathbb{N}$	$\mapsto n \in \mathbb{N}$
(3×)	14, 15, 19	$=$	$\mapsto \equiv$
182	–8	m is solvable	$\mapsto \lceil m \rceil$ is solvable
184	11	KH_4	$\mapsto \mathbf{KH}_4$
	15	$\mathbf{K}^3\mathbf{I}$	$\mapsto \mathbf{K}^4\mathbf{I}$
	–4	16.3.15	\mapsto 17.3.15
	–2	S	$\mapsto \mathbf{S}$

Chapter 9			
186	14	$M \in \Lambda$	$\mapsto \lambda x.M \in \Lambda$
	-5, -4	I	$\mapsto \mathbf{I}$
193	-8	N_m	$\mapsto N_n$
198	-2 (2×)	$\#$	$\mapsto \sharp$
▶ 201	-2	$\forall k \leq \text{lh}(\alpha) \alpha(2k) \leq m$	$\mapsto \forall k \leq n \alpha(2k) \leq m$
203	19	$k = m + k_0$	$\mapsto k = m + k_0 + 1$
	-8	$MI^{\sim p}$	$\mapsto MI^{\sim p+1}$
	-8	\in	$\mapsto \in_\beta$
205	-16	P_{n+1}, \dots, P_p	$\mapsto P_{n+1} \dots P_p$
208	2	\supset	$\mapsto \vec{\supset}$
209	-6	9.1.7	\mapsto 9.1.6
211	14, 16 (2×)	P_p	$\mapsto P_{p'}$
Chapter 10			
216	-6	Böhm tree	\mapsto Böhm tree
220	-4, -3 (2×)	\uparrow	$\mapsto = \perp$
222	20	$i > m$	$\mapsto i \geq m$
	-3	$M_{\langle 0 \rangle} = \mathbf{\Omega}$	$\mapsto M_{\langle 0 \rangle} \equiv \mathbf{\Omega}$
223	-3	\uparrow else.	$\mapsto \perp$ if $\text{lh}(\alpha) = k$ and $\alpha \in A$; $\uparrow\uparrow$ else.
226	15	$\#B_\alpha$	$\mapsto \sharp B_\alpha$
227	-9	\vec{x}_α, y_α	$\mapsto \vec{x}_\alpha, y_\alpha, m_\alpha$
228	1	THEOREM.	\mapsto THEOREM (Bergstra and Klop [1980]).
	-12	$\ulcorner \alpha * \langle 0 \rangle \urcorner$	$\mapsto \ulcorner \alpha * \langle 0 \rangle \urcorner$
	-11	$\#B_\alpha$	$\mapsto \sharp B_\alpha$
▶	-3	$\forall \alpha \{ \vec{x}_\alpha \} \subseteq \bigcup \{ \text{FV}_A(\beta) \mid \beta > \alpha \}$	$\mapsto \forall \alpha [A(\alpha) \Rightarrow \{ \vec{x}_\alpha \} \subseteq \{ y_\alpha \} \cup \bigcup \{ \text{FV}(A_{\alpha^*(i)}) \mid 0 \leq i \leq m_\alpha \}]$.
229	-5	PROOF.	\mapsto PROOF. (i)
230	6	tree topology	\mapsto tree topology
	7	$BT : \Lambda \rightarrow \mathfrak{B}$	$\mapsto BT : \Lambda \rightarrow \mathfrak{B}$
	-9 (3×)	BT	$\mapsto BT$
231	-5	$\lambda x.x$	$\mapsto \langle \lambda x.x, 0 \rangle$
232	10.2.9	BT	$\mapsto BT$
233	1	$A : X$	$\mapsto A; X$
▶ 236	-5	$x_n \leq_\eta A_m$	$\mapsto x_n \leq_\eta A_m \ \& \ x_n \neq y$
▶ 237	-4	$(M; X_\alpha)_\alpha$	$\mapsto M(BT(M); X_\alpha)_\alpha$
238	-11	Σ_1	$\mapsto \Sigma_1 \times \mathbb{N}$
245 – 246 (3×)		BT	$\mapsto BT$
247	5	$(\lambda \vec{x}.M)\vec{N}^*$	$\mapsto (\lambda y \vec{x}.M)P\vec{N}^*$

Chapter 10 (continued)

		$= P_2I(P_2Iy\Omega)$	\mapsto	
248	2			
249	11	U_j^n	\mapsto	U_j^m
	13	$xM_1 \dots M_m$	\mapsto	$xM_0 \dots M_m$
250	-5, -4 (2 \times)	\vec{P}	\mapsto	\vec{R}
	-2	(1)	\mapsto	(i)
251	9	$M \beta = N \beta$	\mapsto	$M \beta \sim N \beta$
	18	$\lambda\vec{x}.yN_{i_1} \dots N_{i_m}$	\mapsto	$\lambda\vec{x}.yM_{i_1} \dots M_{i_m}$
	-6	10.3.6 (ii).	\mapsto	10.3.6 (ii), that also holds for virtual nodes.
253, 254	-13, 5	along	\mapsto	up to
254	-6	(i) By	\mapsto	(i) It suffices to show this for closed P, Q . By
256	4	\simeq_η	\mapsto	$\bar{\simeq}_\eta$
	7	along	\mapsto	up to
257	3	\mathcal{F} if	\mapsto	\mathcal{F} is
	4	10.2.31	\mapsto	10.2.13
258	-9, -6	P_4	\mapsto	P_4
261	2	$\lambda\eta \vdash$	\mapsto	$\lambda\eta \vdash$
	-3	(i)	\mapsto	
263	-8	$xM_{11}^k \dots M_{1m_q}^k$	\mapsto	$xM_{11}^q \dots M_{1m_q}^q$
265	10 - 21 (19 \times)	π	\mapsto	π_1
	11, 13 (2 \times)	$k =$	\mapsto	$p =$
	13	$\lambda\vec{y}_i.z_1$	\mapsto	$\lambda\vec{y}_1.z_1$
	16	$k >$	\mapsto	$p >$
	21, 22 (2 \times)	Max	\mapsto	min
▶	265	26		Before 10.5.21 add the following definition. 10.5.20A. DEFINITION. (i) π is called \mathcal{F} -nonconfusing $\iff \forall M, N \in \mathcal{F}. [M \sim N \iff M^\pi \sim N^\pi]$ & $[M \langle \rangle \downarrow \iff M^\pi \text{ solvable}]$ (ii) $\mathcal{F}_I^o = \{\pi \in I \mid \pi \text{ is } \mathcal{F}\text{-nonconfusing and } M^\pi \text{ is solvable}\}$.
266	2	b_p	\mapsto	b_q
	-12, 11, -9	b_p	\mapsto	b_q
▶	267	\mathcal{F} -faithful	\mapsto	\mathcal{F} -nonconfusing
▶	8, 14, 15, 18,	\mathcal{F}_I^*	\mapsto	\mathcal{F}_I^o
	-6, -1 (6 \times)			
▶	268	\mathcal{F}_I^*	\mapsto	\mathcal{F}_I^o
272	15	$x \in \text{BT}(Fx)$	\mapsto	$x \in \text{BT}(Fx)$, i.e. $x \in \text{FV}(\text{BT}(Fx))$,
	16	$\lambda \vdash$	\mapsto	$\lambda \vdash$
	15	$\langle \text{something like} \rangle \Lambda_1\mathcal{B}$	\mapsto	$\Lambda_1\mathcal{B}$

Chapter 11			
279	2	$\dots(P_2[x_2 := (\dots\Delta'_1\dots)]) \mapsto \dots(P_2[x_2 := (\dots\Delta'_1\dots)])\dots$	
282	2-nd diagram	Arrow from M^\sim to N^\sim should have as label β' (not β).	
285	5	$\{\Delta \mid \Delta \in P$	$\mapsto \{ \Delta \mid \Delta \in P$
287	-11	an <i>weighting</i>	\mapsto a <i>weighting</i>
288	-1	274	\mapsto 278
289	-7	$\lambda_i x_2.P_0$	$\mapsto \lambda_j x_2.P_0$
291	4	$\{M \mid M \xrightarrow[\text{dev}]{\Rightarrow} N\}$	$\mapsto \{N \mid M \xrightarrow[\text{dev}]{\Rightarrow} N\}$
	-4	M	$\mapsto (M, \mathcal{F})$
292	-11	$\omega = \lambda x.xx$	$\mapsto \omega \equiv \lambda x.xx$
	-4	$\vec{1}$	$\mapsto \vec{1}$
294	5	$M' \rightarrow_{\beta_0} N'$	$\mapsto M' \rightarrow_{\beta_0} N'$
296	-3	$\sigma : M \rightarrow N$	$\mapsto \sigma : M \twoheadrightarrow N$
298	-3	$M' \xrightarrow[1,i]{\twoheadrightarrow} M$ even	\mapsto even $M' \xrightarrow[1,i]{\twoheadrightarrow} N$
300	-12 - -4	Argument can be simplified by distinguishing $N \equiv \lambda x.N_0$ and $N \equiv N_0N_1$.	
	-2 (2 \times)	\vec{M}	$\mapsto \vec{P}$
	-1	\vec{N}	$\mapsto \vec{Q}$
	-1	$M_i \twoheadrightarrow N_i$	$\mapsto P_i \twoheadrightarrow Q_i$
301	2, 4, 7 (3 \times)	\vec{N}	$\mapsto \vec{Q}$
Chapter 12			
302	6, 7 (2 \times)	\rightarrow	$\mapsto \twoheadrightarrow$
303	9	12.1.1	\mapsto 12.1.1A
303	16	R -reduction	\mapsto <i>R</i> -reduction
304	2	\cong	$\mapsto \cong$
305	7	D_4	$\mapsto D_3$
	1 st diagram	$\sigma \rho, \rho \sigma$	$\mapsto \sigma/\rho, \rho/\sigma$ (respectively)
310	-7	Since by (2)	\mapsto Since by (3)
316	-4	\cong an	$\mapsto \cong$ is an
319	-1	Δ_{n-1}	$\mapsto \Delta_{n+1}$
320, 321	1, 5	lemma 2.1.12	\mapsto proposition 2.1.12
322	1	(σ)	$\mapsto (\sigma_k)$

Chapter 13

324	-7	effective	\mapsto	effective ³
325	-7	β	\mapsto	β
327	-3	=	\mapsto	\equiv
328	-5	β	\mapsto	β
329	Fig. 13.3.	\mathbb{N}'	\mapsto	\mathbb{N}
330	3	=	\mapsto	\equiv
331	1	reduction	\mapsto	<i>reduction</i>
	-4	M_1	\mapsto	M
	Fig. 13.6	The gk-arrows should be dotted.		
332	7	PROOF.	\mapsto	PROOF. (i)
334	4, 5 (2 \times)	\mathcal{C}	\mapsto	\mathcal{C}_1
▶	4	\twoheadrightarrow	\mapsto	=
338	-1	$0 \leq i \leq n$	\mapsto	$0 \leq i < n$
340	-11 (3 \times)	\rightarrow	\mapsto	\twoheadrightarrow
342	6	order i	\mapsto	order $i - 1$
343	11	13.4.11 (i)	\mapsto	13.4.11 (ii)
	15	13.4.11 (ii)	\mapsto	13.4.11 (i)
	Fig. 13.9 (5 \times)	Arrows labelled ‘cp1’ should have double heads.		
344	1	theorem 11.2.20	\mapsto	proposition 11.2.20
	11	Bergstra-Klop [198+]	\mapsto	Bergstra-Klop [1982]
▶	345	-6, -4 (3 \times)	Replace $\ M\ , \ N\ $ by $\ M\ _1, \ N\ _1$, respectively, where $\ \cdot\ _1$ is defined in the proof of 13.3.5.	
▶		-1	in $\Lambda_{\ M\ }$. \mapsto in $\Lambda_{\ M\ _1}$ and does not contain an F -cycle.	
▶	346	1, 2	Between ‘otherwise’ and ‘ F ’ insert: ‘at least one of the following two situations holds: 1. the F -path of M contains an F -cycle; 2. the F -path of M does not lie within $\Lambda_{\ M\ _1}$. Case 1 is impossible as F is a B -optimal normalizing strategy. If case 2 holds, then’	
	-9	AB	\mapsto	AB
347	5	I	\mapsto	I
	9	l -1-optimal	\mapsto	L -1-optimal
	-3	$B(\Pi) =$	\mapsto	$B(\Pi)) =$

³Some readers prefer the word ‘efficient’.

Chapter 14			
348	5 (2×)	#	↦ #
	11 (2×)	#	↦ #
350	Fig. 13.10	Figure c should be upside down.	
351	-2	than	↦ then
355	17	$\rightarrow_{lab.\beta}$	↦ $\rightarrow_{lab.\beta}$
356	15	$L_1 = xV_1 \dots V_M$	↦ $L_1 \equiv xV_1 \dots V_M$
	-5	$M = M_1M_2$	↦ $M \equiv M_1M_2$
358	7	$(x\vec{N})^* \equiv \perp\vec{N}$	↦ $(x\vec{N})^* \equiv \perp\vec{N}^*$
360	4	an alternative proof	↦ a proof
361	-9	$k = 0$	↦ $k \in \{0, 1\}$
	-9	$k > 0$	↦ $k > 1$
362	-5	14.2.8	↦ 14.2.7
363	2	theorem	↦ proposition
	5, 6	elementary and diagram	↦ elementary diagram
	-11	Δ'_j	↦ Δ'_i
Chapter 15			
364	-7	$\beta(\perp)$	↦ $\beta\perp$
369	8	$P \equiv D[\perp, \dots, \perp]$	↦ $P \beta\leftarrow D[\perp, \dots, \perp]$
	10	$C[\vec{P}] \equiv C[D[\vec{\perp}]]$	↦ $C[\vec{P}] \beta\leftarrow C[D[\vec{\perp}]]$
371	15	lemma 14.3.15	↦ lemma 14.3.14
372	8, 10 (2×)	$C[M] = N$	↦ $C[M] \equiv N$
373 – 374	(9×)	Replace superscripts (n) and (m) by $[n]$, $[m]$, respectively.	
382	16	(i) Suppose	↦ Suppose
	16	redex. Show	↦ redex. (i) Show
	-8	$M \in A$	↦ $M \in \Lambda$
385	10 (2×)	\equiv	↦ $=$
	-5	(3) and (1)	↦ (2) and (1)
	-2	$(xP_1 \dots P_n)^\eta$	↦ $(\dots (x \dots P_1) \dots P_n)^\eta$
386	-6	redexes	↦ β -redexes
387	-4	$\lambda x_1 \dots x_n. x_1 N_2 \dots N_n$	↦ $\lambda x_n \dots x_1. x_1 N_2 \dots N_n$
388	6	$NL_1 \dots L_n$	↦ $NL_n \dots L_1$
	7	$y_2 P_{11} \dots P_{1k_1}$	↦ $y_2 P_{21} \dots P_{2k_2}$
	-9	$M \neq \Omega$	↦ $M \neq \Omega$
	12 – 19 (5×)	Ω	↦ Ω
390	-8	$\downarrow\beta$	↦ $\downarrow\beta$
393	5	$((\lambda x.z(xx))\omega_3)$	↦ $((\lambda x.z(xx))\omega_3)$
395	-10	15.2.4(ii)	↦ 15.2.4(iii)

Chapter 15 (continued)		
396	Fig. 15.4	Interchange labels (1) and (2).
398	1	$\lambda y, M_0 \mapsto \lambda y.M_0$
	4	$H \subseteq M_i \mapsto H \subset M_i$
	-17	$\lambda x_m \cdot y \mapsto \lambda x_m.y$
400	-10	$\delta_C \mapsto \delta_C$
401	6	$\Lambda \delta \mapsto \Lambda^\circ \delta$
403	-3	$M_1, \dots, M_n \mapsto M_1 \dots M_n$
	-2	$\text{BT}(M_1) \text{ BT}(M_n) \mapsto \text{BT}(M_1) \dots \text{BT}(M_n)$
406	diagram	Left arrow with β should be doubly headed.
Chapter 16		
413	-10	$M \approx N \mapsto M' \approx N'$
416	-10	19.2.12 \mapsto 19.2.9
418	-7	19.2.12 \mapsto 19.2.9
419	-11	$\pi_n \mapsto \pi_n$
420	-12	$M \not\approx N' \mapsto M' \not\approx N'$
	-3	$\simeq_\eta \mapsto \widetilde{\simeq}_\eta$
422	2	$\Theta \mapsto \Upsilon$
	2	$B \rightarrow \mapsto Bx \rightarrow$
425	-10, -9 (4 \times)	$BT \mapsto \text{BT}$
426	in fig. (2 \times)	$BT \mapsto \text{BT}$
427	3	$\omega \mapsto \omega$
429	2	14.4.5 \mapsto 15.1.5
	14	$\Theta(jxy.x(jy)) \mapsto \Theta(\lambda jxy.x(jy))$
429	17	$\sqsubseteq \mapsto \sqsupseteq$
	-8	$\forall z \in \Lambda^\circ \mapsto \forall Z \in \Lambda^\circ$
	-5	$AZ = AZ \mapsto AZ = AZ'$
	-4	$AZ = A'Z \mapsto AZ = AZ'$
430	9	$M \subseteq N \mapsto M \sqsubset N$
▶	-4	$J = \lambda + \text{l} = \Omega_3 \equiv \omega_3 \omega_3 \mapsto J = \lambda + \text{l} = \Omega \text{ and } \Omega_3 \equiv \omega_3 \omega_3$
	-3	$=_J \mapsto =_J$
Chapter 17		
▶	435 -9, -10	$O_M = \{N \mid FM \in O\} \mapsto O_M = \{N \mid FN \in O\}$
	443 4	15.1.9 \mapsto 16.1.9(ii)
	465 11	17.1.9 (ii) \mapsto 17.1.9

Chapter 18		
468	4	Palamidessi Catuscia \mapsto Catuscia Palamidessi
473	-6	$\lambda^G x.A$ \mapsto $\lambda x.A$
▶ 476	12	$(\lambda x.A)\emptyset = \emptyset$ \mapsto $(\lambda x.A)\emptyset \neq \emptyset$
	-2	$e_k \not\subseteq e_k$ \mapsto $e_k \not\subseteq e_{k'}$
477	-2, -1 (2 \times)	$\psi(x)$ \mapsto $\psi(x')$
481	12	$\langle \sqcup_n x_{\min}(n, i) \rangle_{i \in \mathbb{N}}$ \mapsto $\langle \sqcup_n x_{\min}(n, i) \rangle_{i \in \mathbb{N}}$
483	11	$y \in_n$ \mapsto $y \in D_n$
485	-3	$\Phi_{n,\infty}(\lambda y \in D_n)$ \mapsto $\Phi_{n+1,\infty}(\lambda y \in D_n)$
491	18.4.1	$P\omega \forall \mathbf{1} = \mathbf{l}$ \mapsto $P\omega \not\equiv \mathbf{1} = \mathbf{l}$
492	-10	15.3.4 \mapsto 15.4.4
Chapter 19		
508	1	19.2.14 \mapsto 19.2.11
509	-6	operators \mapsto combinators
	-5	combinator \mapsto operator
Chapter 20		
513		5.3.25 \mapsto 5.2.23(i)
518	12	Wadsworth. \mapsto Wadsworth. Write $x \in \text{BT}(M)$ for $x \in \text{FV}(\text{BT}(M))$.
521	-5	19.3.15 \mapsto 18.3.15
Appendices		
570	11	[198+] \mapsto [1982]
576	-3	[1980] \mapsto [1979]
581	6 (2 \times)	v_2 \mapsto v_1
584	20	[198-] \mapsto [198?]
Addenda		
582	6	$xz(yx)$ \mapsto $xz(yz)$

References		
585	-4	[198-] \mapsto [1983]
	-3	to appear. \mapsto 48, pp. 931-940
586	-22	[1982] \mapsto [1982a]
	-11	Add: BEZEM, M. A. [1985] Isomorphisms between HEO and HRO^E , ECF and ICF^E . <i>Journal of Symbolic Logic</i> , 50 , pp. 359-371.
	-8	[1980] \mapsto [1979]
589	15 (2 \times)	[1980] \mapsto [1979]
595	5, 6	(to appear). \mapsto no. 3, pp. 271-286, 287-302, 303-325.
597	-9	\mathbf{p} -functions \mapsto \mathbf{p} -function
598	-8	AGM \mapsto ACM
Index of Names		
599	$R17$	Add: Bezem, M. [1985] 566
	$L-19$	516 \mapsto 250, 516
	$R-16$	238, \mapsto 238, 245,
600	$R13$	[1980] \mapsto [1979]
601	$R21$	116 \mapsto 116,107
	$R22$	[1983] 107 \mapsto [1984] 494
602	$R11$	149 \mapsto 150
	$L-17$	536 \mapsto 534
	$L-13$	75 \mapsto
	$L-11$	O'Donell \mapsto O'Donnell
Index of Definitions		
607		leftmost 179 \mapsto leftmost 180
608	$L12$	\mapsto Paradox Curry- 573, 575 Liar's- 573
Index of Symbols		
613	11	λI -terms \mapsto λI -terms
614	8	$M\check{\emptyset}$ \mapsto $M\check{\emptyset}\vec{N}\check{\emptyset}$
	19	354 \mapsto 355
618		Add: $A =_{\eta} B$ trees A, B are equal up to (possibly 240 infinitely many) η -conversions
619	-15	150 \mapsto 160
620 (2 \times)	11, 12	$\Vdash M = N$ \mapsto $\models M = N$

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