

Exercises week 7

Languages and Automata

June 14, 2012

7. More on context-free languages

- 7.1. Let the alphabet $\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, -, +, \times, \cdot, \cup, \cap, \setminus, \emptyset, \lambda, \epsilon, \dots\}$ be given.
- Give a context free grammar that exactly accepts the expressions denoting integers (like 0, -12, 713, ...). Avoid integers like '007'; this one should be denoted as just '7'.
 - Give a context free grammar that accepts integer expressions with +, - and \times . (Like '(1 + -(3 \times 22))' and '22 \times (1 - 3)', ...).
 - Prove with the pumplemma that the grammar in (ii) isn't regular.
- 7.2. Given grammar G_1

$$\begin{aligned} S &\rightarrow aABb \\ A &\rightarrow aA|a \\ B &\rightarrow bB|b \end{aligned}$$

and grammar G_2

$$\begin{aligned} S &\rightarrow AABB \\ A &\rightarrow AA|a \\ B &\rightarrow BB|b \end{aligned}$$

- Construct a PDA accepting $L(G_1)$.
 - Prove $L(G_1) \subseteq L(G_2)$
 - Prove $L(G_2) \subseteq L(G_1)$
- 7.3. Let $\Sigma = \{a, b\}$. Define $\Sigma^+ = \{\emptyset, \lambda, a, b, \cup, *, \cdot, \setminus, \emptyset, \lambda, \epsilon, \dots\}$. The set R_Σ of regular expressions over Σ are special words over Σ^+ , for example

$$\begin{aligned} (a \cup \lambda) \cdot (b \cdot a^*)^* &\in R_\Sigma, \\ (a \cup \cup b) &\notin R_\Sigma. \end{aligned}$$

- Show that R_Σ is context-free.
- Show that R_Σ is not regular. [Hint. Apply the pumping lemma for regular languages.]