Test Typed lambda calculi 2006 **October 12 2006**

1. Let $\mathsf{I} \equiv \lambda y.y,\ V \equiv \lambda x.\mathsf{I}xx,\ M \equiv VV.$
Construct the reduction graph of M, that is find all possible beta-reducts of M connected by a directed arc if there is a beta one step reduction between them.[Hint. There are 8 nodes and 15 arcs.]

2. Let $1_3 = (0{\to}0{\to}0{\to}0)$ and $2 = ((0{\to}0){\to}0)$.

   Find all inhabitants in normal form of the types
   $1_3{\to}0{\to}0$ and $2{\to}0{\to}0$.

   You may describe the collections of inhabitants schematically or in words or in any other manner, as long as it is clear.
   Remember that a term in normal form has the form
   $\lambda x_1 \ldots x_n.xM_1 \ldots M_m \quad n, m \geq 0.$

3. Find types for the terms $\lambda xyz.xzzy,\ \lambda xy.x(x\mathsf{I}y)$, with $\mathsf{I} \equiv \lambda z.z$.