Test 3 Typed lambda calculi 2006   **December 13 2006**

1.   Prove that $1_2 \leq_{\beta\eta} 1_3$ by defining

$$\Phi : 1_2 \to 1_3 \text{ and } \Psi : 1_3 \to 1_2$$

such that
$$\Psi(\Phi(M)) =_{\beta\eta} M.$$

2.   Determine $[\![\lambda x.x]\!]_\rho^{\mathcal{F}_s^{CD}}$ for all $\rho : Var \to \mathcal{F}_s^{CD}$.
      This can be done by first principles, not using the Type-semantics Theorem.

3.   This exercise is on the type system BCD and the assignment system $\lambda_{\cap^\top}^{BCD}$.
      α and β are different type variables.
      (i)   Let $\mathcal{E}$ be the class of types inductively defined by

$$\mathcal{E} = \alpha \,|\, \top \,|\, (C \to D \text{ where } D =_{BCD} \top) \,|\, \mathcal{E} \cap \mathcal{E}.$$

      Prove

$$(B \in \mathcal{E} \;\&\; B \leq A) \Rightarrow A \in \mathcal{E}.$$

      (ii) Prove
$$\nvdash_{\cap^\top}^{BCD} \lambda x.xx : \alpha \to \beta.$$