Reflection and its use

from science to meditation

Henk Barendregt
Katholieke Universiteit Nijmegen

Music: Miserere by Gregorio Allegri (1582?-1652)

Composed before 1638

Papal monopoly to perform it only at the Vatican (during Easter)

Copying the music would be punished with excommunication

In 1770 the 13 year old Mozart heard it on a trip in Italy

Made a copy in his hotel from memory

The noted music bibliographer and historian Dr. Burney from London met Mozart that year (August) in Bologna

Probably in this way the music was published
in England, Paris, Leipzig and Rome (1771)

Going from score to music is rare

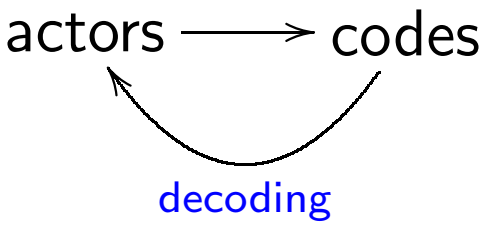
usually it is the other way around

Reflection

Actors: active objects

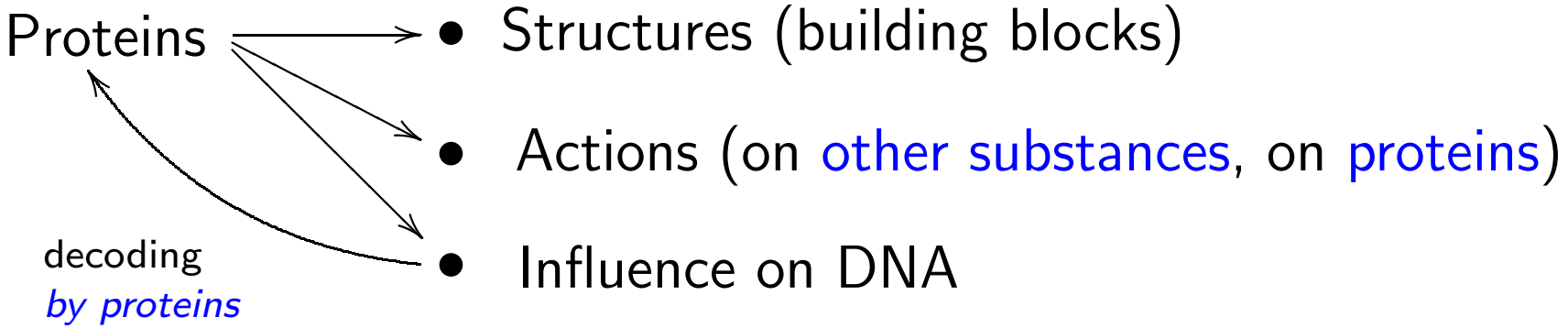
Codes: usually passive; decoding $\longrightarrow$ actors

Interaction:



Topic	Actors	Codes
Biology	proteins	DNA (RNA)
Language	sentences	'citations'
Mathematics	statements	Gödel numbers
Computing	computable functions	programs
Meditation	consciousness	mindfulness

Reflection in biology



Proteins influence DNA for

- corrections
- duplication
- decoding

and therefore have influence on themselves as population

(essential for life)

Reflection in language

Language is active

She: Will you visit me tonight?
He: Yes. Unless it rains.

Language has a coding-mechanism (Tarski [1933])

- Maria is a nice girl
- 'Maria' consists of five letters

Interaction

> Mother, what is the meaning of the word 'curious'?

Important for language acquisition, analysis and extension

The sentence 'I am sad' has by itself no emotional value
I did not utter it

Reflection in mathematics

One may consider mathematical expressions

not for their *meaning*
but for their *syntactic form*

$\longmapsto$ stronger results

For quantitative expressions: algebra

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

For qualitative expressions:

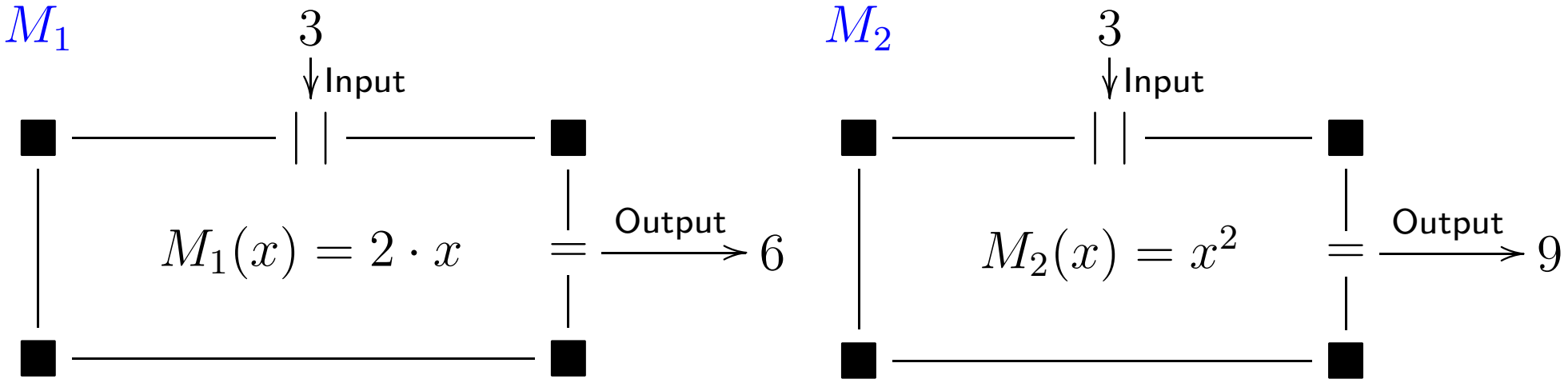
$$\text{The function } \tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} \text{ is continuous}$$

Metamathematics: Gödel's theorem

1. Arithmetic speaks about numbers
2. (Pythagoras) Everything is a number (after coding)
3. Arithmetic speaks about everything you want (via coding)
4. Arithmetic speaks about arithmetic
5. So arithmetic speaks about itself(!)
6. L: This statement is false
7. G: This statement is unprovable from the Peano axioms
8. If PA is free from contradictions, then G is not provable and hence true(!)

Moral: Arithmetic is incomplete

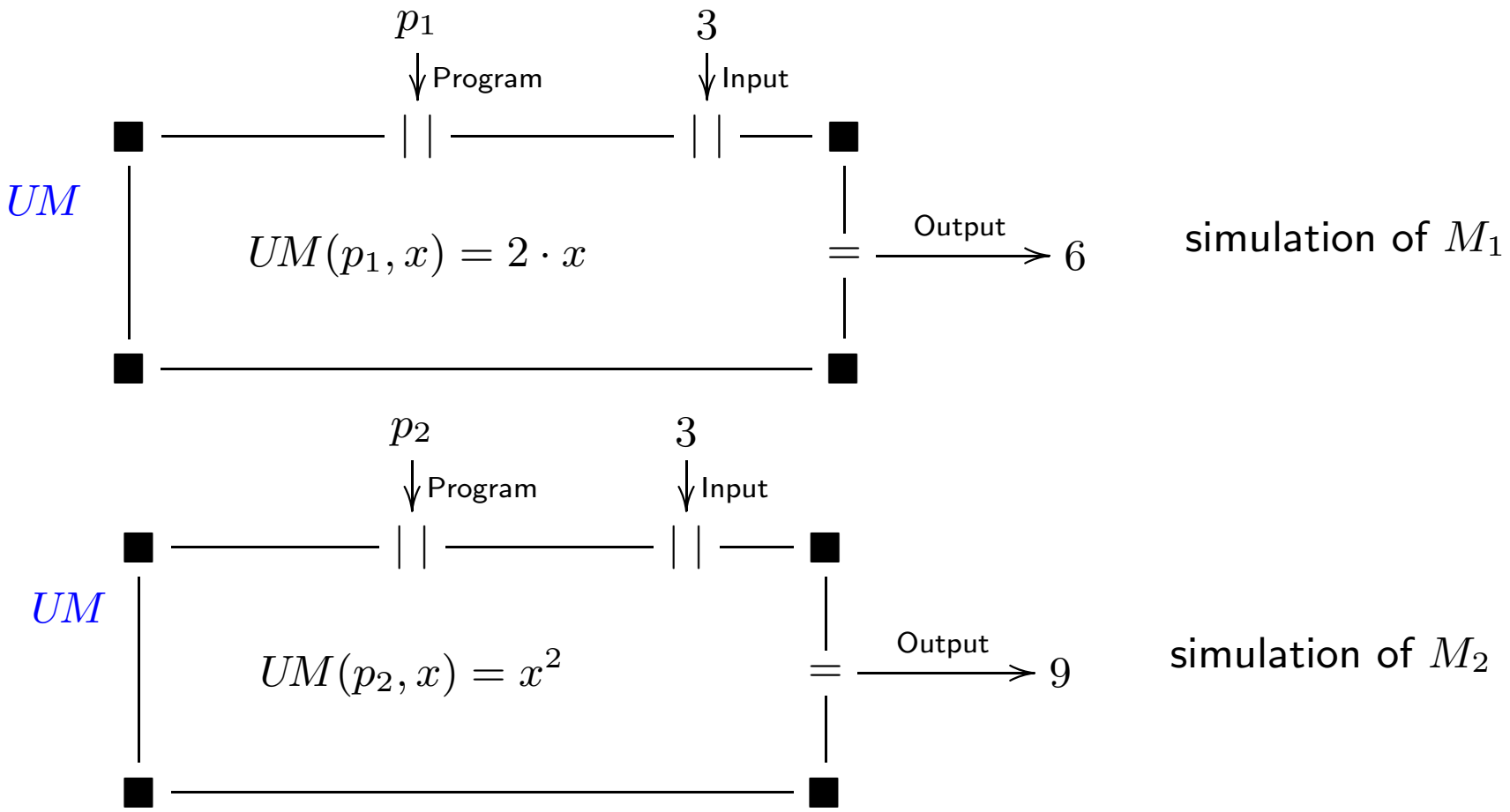
Reflection in computers



Two *ad hoc* machines:

M_1 for doubling and M_2 for squaring

Reflection in computers



One Universal Machine UM with programs simulating M_1 and M_2
M_2 can act on the code of M_1 or on that of itself:

$$\boxed{M_2(p_1) \text{ of } M_2(p_2)}$$

Reflection in computers

Computable functions getting their own program as input

is important for the so-called *bootstrap*

Reflection in consciousness

reality	consciousness
	consciousness (of colour)
consciousness (of colour)	consciousness of consciousness

‘mindfulness’

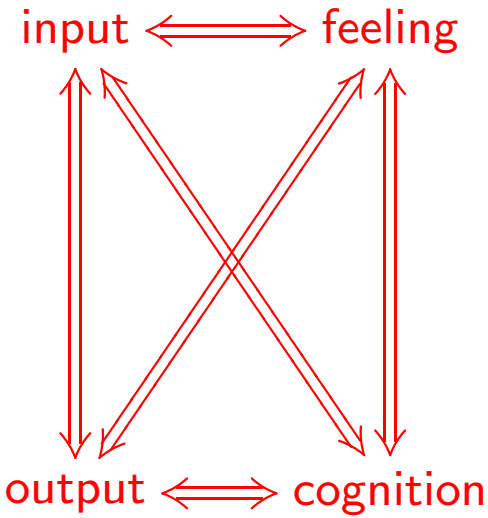
action $\longmapsto$ sadness
sadness $\longmapsto$ consciousness of sadness

Reflection in consciousness

Active contents of consciousness

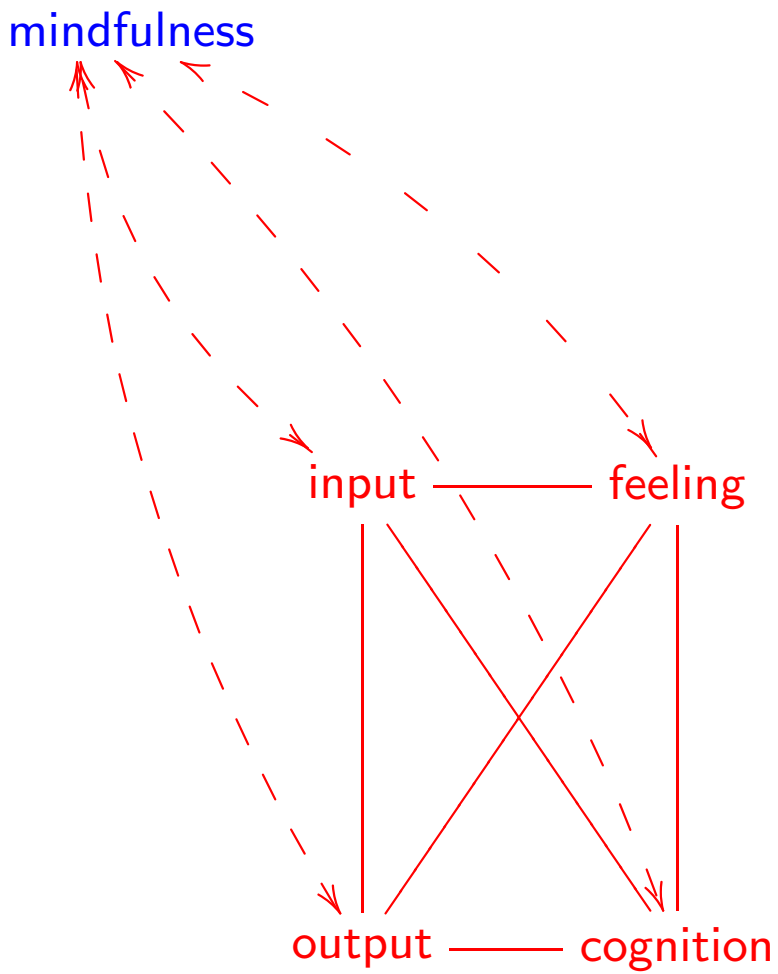
$\mapsto$ *Conditioning*

Spinoza described this problem
his solution: ethics



Duty moral

Reflection in consciousness



Mindfulness is non-active

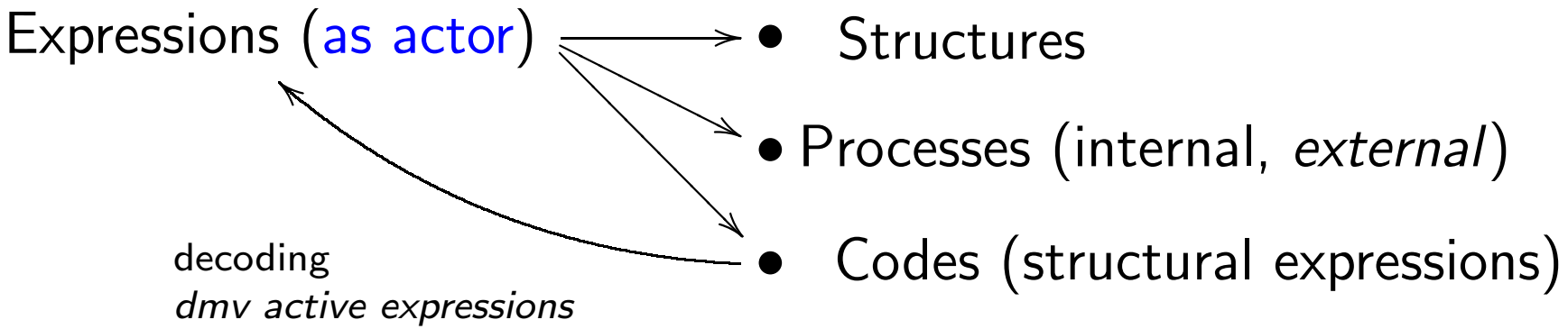
$\mapsto$ less conditioning

Virtue moral

Lambda calculus: theory of reflection

Expressions & computation rules

Expressies: actors (external of internal) through the rules



$$(\lambda x.x \times x)\ 3 \quad \twoheadrightarrow_\beta \quad 3 \times 3$$
$$\twoheadrightarrow_\iota \quad 9$$

Lambda calculus: theory of reflection

Pure: without external structures and processes

Applied: with external structures and processes

- evolutiontheory
- linguistics
- mathematics
- computable functions

$\lambda \longmapsto$ intuitionistic logic and mathematics $\longmapsto$ Computer Mathematics

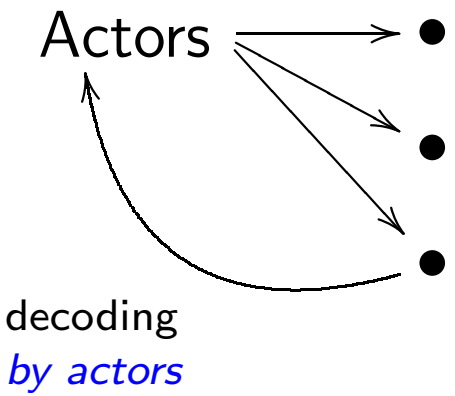
Representation of arbitrary mathematical objects on a computer
by means of verified formal proofs

Aim: development of a powerful mathematical assistant

existing mathematics	learning, teaching, applying
new mathematics	developping

Reflection summarized

Global feedback



Structures

Processes (external, internal)

Codes

Power and draw-back of reflection

Topic	'negative' effect	'negative' effect
Biology	life	virusses
Language	homo sapiens	paradoxes
Mathematics	science	incompleteness
Computing	IT	undecidability
Meditation	purification	dissociation

Negative effects of reflection

Biology	virusses	code plus auxiliary protein
Language	paradoxes	'This sentence is false'
Mathematics	incompletess	provability $\neq$ truth
Computing	undecidability	not all problems can be decided
Mental reflection	dissociation	unity of self falls apart

Number-like versus language-like data

Measuring has been a useful act in science

For many centuries people wondered what gravity is

Galileo measured the position of a sliding marble over time

That was the key to real understanding

One cannot always describe phenomena by (floatingpoint) numbers

In the sciences many things are “language-like”

For example the domains of reflection

Allowing large numbers the two approaches come together

The color controversy (Newton vs Goethe)

Physicists: colors can be described by one number (wavelength)

Phenomenologists: colors cannot be described by one number.

Goethe did interesting experiments with perception

Follow up by Land in the 1950's

See <www.physicstoday.org/vol-55/iss-7/p43.html>.

Colors I

(.99,.99,.99)

(.99,.99,.78)

(.99,.99,.50)

(.99,.99,.00)

(.99,.78,.99)

(.99,.78,.78)

(.99,.78,.50)

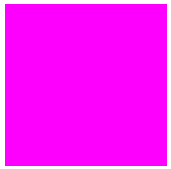
(.99,.78,.00)

(.99,.50,.99)

(.99,.50,.78)

(.99,.50,.50)

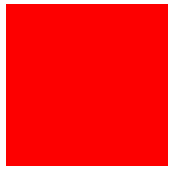
(.99,.50,.00)



(.99,.00,.99)

(.99,.00,.78)

(.99,.00,.50)



(.99,.00,.00)

Colors II

(.78,.99,.99)

(.78,.99,.78)

(.78,.99,.50)

(.78,.99,.00)

(.78,.78,.99)

(.78,.78,.78)

(.78,.78,.50)

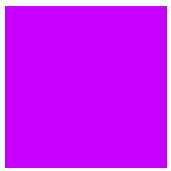
(.78,.78,.00)

(.78,.50,.99)

(.78,.50,.78)

(.78,.50,.50)

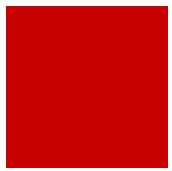
(.78,.50,.00)



(.78,.00,.99)

(.78,.00,.78)

(.78,.00,.50)



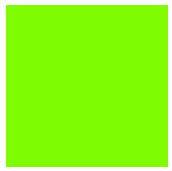
(.78,.00,.00)

Colors III

(.50,.99,.99)

(.50,.99,.78)

(.50,.99,.50)



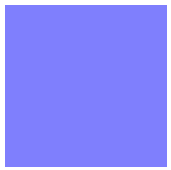
(.50,.99,.00)

(.50,.78,.99)

(.50,.78,.78)

(.50,.78,.50)

(.50,.78,.00)

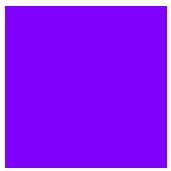


(.50,.50,.99)

(.50,.50,.78)

(.50,.50,.50)

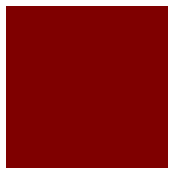
(.50,.50,.00)



(.50,.00,.99)

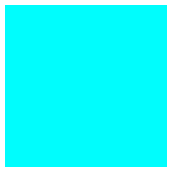
(.50,.00,.78)

(.50,.00,.50)

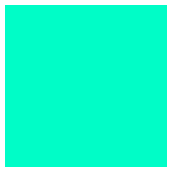


(.50,.00,.00)

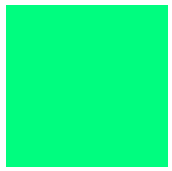
Colors IV



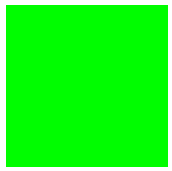
(.00,.99,.99)



(.00,.99,.78)



(.00,.99,.50)

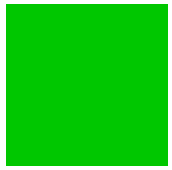


(.00,.99,.00)

(.00,.78,.99)

(.00,.78,.78)

(.00,.78,.50)

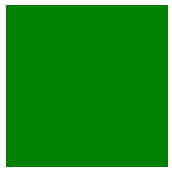


(.00,.78,.00)

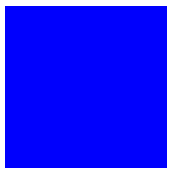
(.00,.50,.99)

(.00,.50,.78)

(.00,.50,.50)

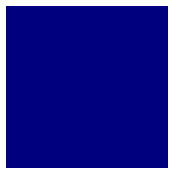


(.00,.50,.00)



(.00,.00,.99)

(.00,.00,.78)



(.00,.00,.50)

(.00,.00,.00)

The color controversy resolved (Young and Helmholtz)

The retina has rods and cones for vision

- rods for black and white vision
- cones for colorvision

There are three kinds of cone-cells

The sensitivity for the wavelength is different

There are the b-type, the g-type, the r-type of cone-cells

Each wave-length creates a triplet of reactions in these three cells

Moral: phenomenology is important

It should be backed by experiments and theory

Colors I

rgb (.99,.80,.50,.00)

(.99,.99,.99)

(.99,.99,.80)

(.99,.99,.50)

(.99,.99,.00)

(.99,.80,.99)

(.99,.80,.80)

(.99,.80,.50)

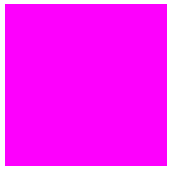
(.99,.80,.00)

(.99,.50,.99)

(.99,.50,.80)

(.99,.50,.50)

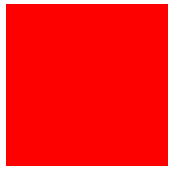
(.99,.50,.00)



(.99,.00,.99)

(.99,.00,.80)

(.99,.00,.50)



(.99,.00,.00)

Colors II

rgb (.99,.80,.50,.00)

(.80,.99,.99)

(.80,.99,.80)

(.80,.99,.50)

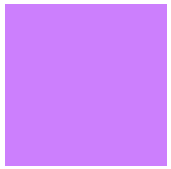
(.80,.99,.00)

(.80,.80,.99)

(.80,.80,.80)

(.80,.80,.50)

(.80,.80,.00)

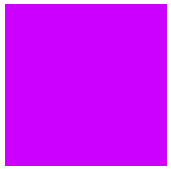


(.80,.50,.99)

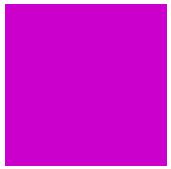
(.80,.50,.80)

(.80,.50,.50)

(.80,.50,.00)



(.80,.00,.99)



(.80,.00,.80)

(.80,.00,.50)

(.80,.00,.00)

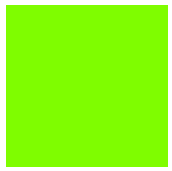
Colors III

rgb (.99,.80,.50,.00)

(.50,.99,.99)

(.50,.99,.80)

(.50,.99,.50)



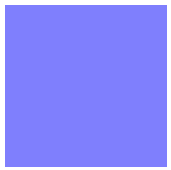
(.50,.99,.00)

(.50,.80,.99)

(.50,.80,.80)

(.50,.80,.50)

(.50,.80,.00)

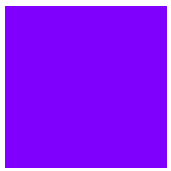


(.50,.50,.99)

(.50,.50,.80)

(.50,.50,.50)

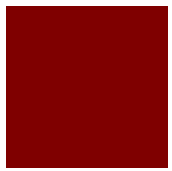
(.50,.50,.00)



(.50,.00,.99)

(.50,.00,.80)

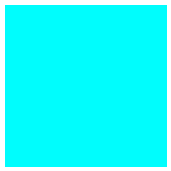
(.50,.00,.50)



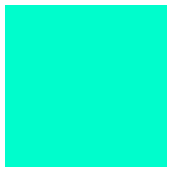
(.50,.00,.00)

Colors IV

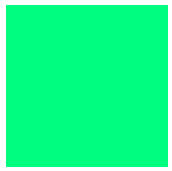
rgb (.99,.80,.50,.00)



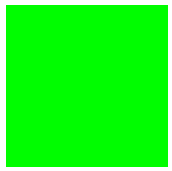
(.00,.99,.99)



(.00,.99,.80)



(.00,.99,.50)

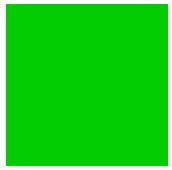


(.00,.99,.00)

(.00,.80,.99)

(.00,.80,.80)

(.00,.80,.50)

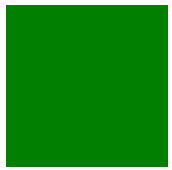


(.00,.80,.00)

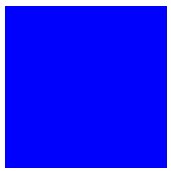
(.00,.50,.99)

(.00,.50,.80)

(.00,.50,.50)

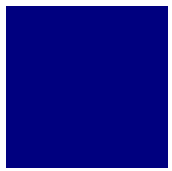


(.00,.50,.00)



(.00,.00,.99)

(.00,.00,.80)



(.00,.00,.50)

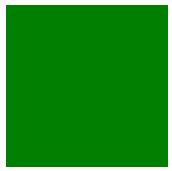
(.00,.00,.00)

Colors IV zoomed-in (rgb) (.50,.30,.15,.00)

(.00,.50,.50)

(.00,.50,.30)

(.00,.50,.15)



(.00,.50,.00)

(.00,.30,.50)

(.00,.30,.30)

(.00,.30,.15)

(.00,.30,.00)

(.00,.15,.50)

(.00,.15,.30)

(.00,.15,.15)

(.00,.15,.00)

(.00,.00,.50)

(.00,.00,.30)

(.00,.00,.15)

(.00,.00,.00)

Colors I zoomed-in (rgb) (.99,.93,.88,.80)

(.99,.99,.99)

(.99,.99,.93)

(.99,.99,.88)

(.99,.99,.80)

(.99,.93,.99)

(.99,.93,.93)

(.99,.93,.88)

(.99,.93,.80)

(.99,.88,.99)

(.99,.88,.93)

(.99,.88,.88)

(.99,.88,.80)

(.99,.80,.99)

(.99,.80,.93)

(.99,.80,.88)

(.99,.80,.80)

Language-like objects

The colors can be described by 3 numbers

The domains of reflection usually have a language-like description

proteins	dna
sentences	citations
statements	Gödel numbers
computable functions	programs
¿consciousness	mindfulness?

Not a language-like object

A splash of water

Continuous geometric figures

But, Feynman:

The most important fact in science:

the world is made from discrete objects

In this way we do not need a mysterious element ‘heat’

In this way we can do chemistry

Nevertheless the oder of continuous things is even larger than the order of language-like things

Music

Tones are produced in an arithmetic sequence

We hear equal intervals as a geometric sequence

production $\lambda,\ 2\lambda,\ 3\lambda,\ 4\lambda,\ 5\lambda,\ 6\lambda,\ 7\lambda,\ 8\lambda,\ \ldots$

hearing $\lambda,\ 2\lambda,\ 4\lambda,\ 8\lambda\ \ldots$

Cf. Law of Weber-Fechner (intensity given and perceived):
equal ratio between inputs gives equal phenomenological differences

The tension between these arithmetical and geometric sequences creates harmony

If $\lambda = 264/\text{sec}$, then

$\lambda,$	$2\lambda,$	$3\lambda,$	$4\lambda,$	$5\lambda,$	$6\lambda,$	$7\lambda,$	$8\lambda,\ \ldots$
C	C'	G'	C''	E''	G''	$\sim B^{\flat\prime\prime}$	$C'''\ \ldots$

Tuning

The twelf notes of the octave (white and black piano keys) are approximately given by a tower of perfect fifths ("quinten")

$$C \quad G \quad D \quad A \quad E \quad B \quad F^\sharp \quad C^\sharp \quad G^\sharp \quad D^\sharp \quad A^\sharp \quad E^\sharp \quad (B^\sharp \sim C)$$

The difference between the $B^\sharp$ and C is called the *comma of pythagoras*

Ravel's *Daphnis and Chloé* starts with the creation of the 8 tones of the octave

$$A \quad E \quad B \quad F^\sharp \quad C^\sharp \quad G^\sharp \quad D^\sharp \quad A^\sharp$$

Rearranged

$$A \quad B \quad C^\sharp \quad D^\sharp \quad E \quad F^\sharp \quad G^\sharp \quad (A)$$

This is the myxolidian scale:

fa, sol, la, si, do, re, mi, fa

Daphnis et Chloé, by Ravel

Defining the alphabet

Painting with music (music like water; also in *Ondine*)

Erotic art

Alphabets

An *alphabet* Σ is a set of symbols

A *word* over Σ is a finite string of elements of Σ

Example

$$\Sigma_{ab} = \{a, b\}$$

Then $abba$ is a word over Σ_{ab}

$abracadabra$ is not a word over Σ_{ab}

Notation

Σ^* collection of words over Σ

$abba \in \Sigma_{ab}^*$

$abracadabra \notin \Sigma_{ab}^*$

Words

Let $\Sigma_{01} = \{0, 1\}$

Then Σ_{ab} and Σ_{01} are *isomorphic*

Enumeration of Σ_{01}^*:

0 elements	''
1 element	$0, 1$
2 elements	$00, 01, 10, 11$
3 elements	$000, 001, 010, 011, 100, 101, 110, 111$
	$\ldots$

The empty string is also denoted by ϵ

In biology the alphabets

$$\Sigma_{acgt} = \{A, C, G, T\} \text{ and } \Sigma_{acgu} = \{A, C, G, U\}$$

play an important role

Languages

Let Σ be an alphabet

A *language* over Σ is a collection L of words in Σ^*

Notation: $L \subseteq \Sigma^*$

The strings of a, b's with

an even number of a's
an odd number of b's

is a language L_{eo} over Σ_{ab}

For example

$$\begin{aligned} abababa &\in L_{eo} \\ ababa, abba &\notin L_{eo} \end{aligned}$$

Hofstadter's MU puzzle

Let $\Sigma_H = \{\mathsf{I}, \mathsf{M}, \mathsf{U}\}$

We generate the following language L_H over Σ_H

axiom	MI
rules	$x\mathsf{I} \Rightarrow x\mathsf{IU}$ $\mathsf{M}x \Rightarrow \mathsf{M}xx$ $x\mathsf{III}y \Rightarrow x\mathsf{U}y$ $x\mathsf{UU}y \Rightarrow xy$

This means that by definition MI in L_H

if $x\mathsf{I}$ in L_H, then also $x\mathsf{IU}$
if $\mathsf{M}x$ in L_H, then also $\mathsf{M}xx$
if $x\mathsf{III}y$ in L_H, then also $x\mathsf{U}y$
if $x\mathsf{UU}y$ in L_H, then also xy

Is the following true or not true:

$$\mathsf{MU} \text{ in } \Sigma_H?$$