

25. Show that $\mathbf{S}(\mathbf{K}\mathbf{I})(\mathbf{II})$ and $(\lambda x. x x)\mathbf{S}$ are typable in \vdash_{\cap}^{CD} .

26. Prove $\nvdash_{\cap}^{CD} \lambda x y. x y : \alpha \rightarrow \alpha$, where α is any constant.

27. We say that M and M' have the same types in Γ , notation $M \sim_{\Gamma} M'$ if

$$\forall A[\Gamma \vdash M : A \Leftrightarrow \Gamma \vdash M' : A].$$

Prove $M \sim_{\Gamma} M' \Rightarrow MN \sim_{\Gamma} M'N$.

28. Prove that Theorem 16.1.10(ii) still holds if the condition of properness is replaced by the following two conditions

$$\begin{aligned} A \leq B &\Rightarrow C \rightarrow A \leq C \rightarrow B. \\ (A \rightarrow B) \cap (C \rightarrow D) &\leq A \cap C \rightarrow B \cap D. \end{aligned}$$

29. Prove

(i) CD is not η -sound and BCD is not $\eta\top$ -sound.

(ii) $\lambda_{\cap}^{CD} \not\models (\eta\text{-exp})$ and $\lambda_{\cap\top}^{BCD} \not\models (\eta\text{-exp})$.