Kreisel, lambda calculus, a windmill and a castle

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To GK

This paper gives some idea of the role that Kreisel played at the start of my scientific career. The facts are taken mainly from the period starting spring 1971 (when I worked on my Ph.D. thesis) until summer 1972 (when I ended a stay at Stanford as a postdoc).

1. The setting

At Utrecht University in the late sixties I studied logic under reader Dirk van Dalen. In those days readers in the Netherlands did not have the *ius promovendi*, i.e. were not allowed to be official Ph.D. supervisors. Kreisel had accepted to spend a spring semester at Utrecht and van Dalen asked whether he was willing to be my Ph.D. supervisor. For this I had to send a description of my work to Kreisel and in a few days the answer was there. The 'master' found my work "congenial" and it could be the basis for discussions leading towards a thesis. Kreisel was supposed to be difficult, so I had heard. Actually this inspired me, while I prepared the manuscript before his coming to Utrecht in 1971.

When Kreisel arrived in the Netherlands, I had borrowed my fathers car (a Renault 16), because mine (a Renault 4) was total-loss due to an accident, and I picked him up from the airport. Kreisel was pleased by this reception 'in style' and for a moment he forgot to be difficult. When we stopped for gas, however, he started to ask sharp questions that were meant to frighten me. Since I did not have a high opinion about myself (I did not have a low one either), it did not matter to me that I could not answer all of his questions. The fact that I was unconditioned by his interrogation mellowed him down and in high spirits we arrived at Utrecht University.

It was well-known that Kreisel always requests a quiet place to sleep. After some effort Mrs. Dook van Dalen had found a candidate place in Bilthoven, a fancy village near Utrecht. But one never knows if it is really adequate. Therefore I showed Kreisel a picture of my home—a windmill in the countryside along the Waal river, the main branch of the Rhine—and offered him to stay there in case he wanted.

After the obligatory inspection of the house in Bilthoven, Kreisel decided to stay in the windmill. This brought Mrs. van Dalen in an awkward situation. The owner of the house had been asked to provide a quiet room for Kreisel. Being sensitive to who was to come into her house, the landlady had emptied her own bedroom on the garden side for the use of the famous professor. From the cool reaction of Mrs. van Dalen about Kreisels decision to stay in the windmill one could deduce the fury of the old lady when her kind offer was declined.

The windmill was situated 45 minutes by car from the University, a relatively long distance in the Netherlands. Arrived there, Kreisel told me that the next day he was invited by a baroness for tea at her castle and asked me to join him. "Of course one of the reasons to ask you is that you can help to drive me there," was his frank admission. By a remarkable coincidence the castle was only 2 kilometer upstream from the windmill. I knew the place, but had no idea who lived there.

Against my expectations the baroness was a young woman, good-looking with two small children. After being introduced to her I felt a bit uneasy about the angle in which she shook hands. Was I supposed to give a handkiss?[1] The baroness was British and lived with her husband in London. Often during weekends the baroness and her children would come to their Dutch castle, while the lord remained in London. In the 15th century ancestors of the baron had been victors of a small battle in the Netherlands. Because of this historical fact his lineage was proclaimed to be 'Baron van Ophemert en Zennewijnen' since that time.

During tea Kreisel invited the lady to come to his first lecture in Utrecht, the following week. She accepted. "It is better for you to leave the lecture-hall after twenty minutes, because then I will start to be rather technical," was Kreisels advice.

On the morning of the day of the first lecture I dropped Kreisel at the castle. He and the baroness were going to have lunch in Utrecht and would come together to the University. At 2:00 p.m. the logicians in the Netherlands were eagerly waiting. At 2:15 Kreisel was still not there. I gave them some details. Nobody yet at 2:30. Finally at 2:40 a taxi entered the parking place of the department,

[1]The next day I borrowed a book on etiquette from a girl-friend in Utrecht. My question was not answered, though.

directly followed by a car with a British license plate. Kreisel came out of the British car and paid the taxi and the baroness parked her car. This scene could be followed by all Dutch logicians from the lecture-hall on the sixth floor of the building. Troelstra remarked that Kreisel was still wearing the same trousers as in Stanford.[2] When at last the lecture started at 2:50, it went as follows. “Logic is the science of deduction. How we can derive from a statement A another statement B. . . . ” This went on for twenty minutes. When the baroness left—as foreseen—she had to pass a door next to the blackboard in front of the audience. Shyly she opened the door. Kreisel, however, was fully at ease. With an elegant bow he paid his respects to the lady. At 3:10 the real lecture started. It was on Rosser sentences, brilliant and full of firework. I could not follow it.[3]

A few days later Kreisel and I were invited by the baroness for dinner at her castle. Impressive stairs went down to a cellar with burning torches on the walls. Some of the dishes were exotic. During the dessert Kreisel showed a side of his, that was new to me. One of the children started to whine about the pudding. “Ma, I do not want the raisins in it; ma, I want you to take them out. . . .” It went on and on. Then Kreisel spoke very slowly to the little one, a boy about six years old: “Listen. What do you think is worse: your mother does not want to take the raisins out; or, your mother is not able to take them out.” The little boy had to think for a while. “It is worse, if she does not want to take them out,” was the answer. “Well,” Kreisel continued, “it is simply the case that she is not able to take them out!” The boy could do nothing else but eat his pudding.

Kreisel and I often visited the castle. Since then the baron also came more frequently to the Netherlands. I was impressed by Kreisels explanations to the lord and his wife on the research I was doing in combinatory logic: “When we reason, we make steps, deductive steps. These can be smaller or larger steps. Barendregt studies the smallest possible such steps—the so-called atomic steps—and the way they can be combined to form larger ones.” The high-born couple understood this, and at the same time I learned to be more flexible when expressing myself.

Also in other places of the world Kreisel introduced me to his ‘upper-class’ friends. The definition of this predicate varied from country to country. Sometimes they were aristocratic refugees from pre-war Europe, sometimes highly successful leaders of industry; but also there were high party members of some—then—powerful political party.

[2] Troelstra had been a visitor at Stanford a few years before.

[3] Later I mentioned this to Kreisel, telling him at the same time that in private conversations I could understand perfectly well his technical remarks. Also I said that I could not understand many of his writings. “Oh, but Gödel can, and Bernays can,” was his reaction.

2. The work

In the early 1970s the λ-calculus was considered to be a fringe area of logic. This in spite of the breakthrough by Dana Scott, who constructed in November 1969 the first set-theoretic model D_∞ of the λ-calculus. Kreisel, who makes a point of being interested in neglected areas of research, showed some genuine interest in the subject. This was very encouraging to me. I will discuss two main themes of my thesis (the ω-rule and (un)solvability) and their later developments.

The ω-rule

For my thesis, Barendregt [1971, 1971a], I wanted to construct a recursion theoretic model of the type-free λK-calculus.[4] Even today I have never been able to complete the construction. But the attempts proved fruitful. One of the candidate models was extensional and a hard one, i.e. generated by the closed terms. This implied the consistency of the following ω-rule.

$$FZ = GZ, \text{ for all closed terms } Z \quad \Rightarrow \quad F = G. \tag{1}$$

Since the model was resisting, I wanted to settle the consistency of this rule by other means. This was done using a proof-theoretic ordinal analysis. Then I wanted to know whether the proof was relevant. Perhaps rule (1) was simply derivable; in that case the consistency is trivial. It turned out that rule (1) was derivable (for $\beta\eta$-reduction), except possibly for some pathological terms F, G, the so-called *universal generators*. This almost proved the validity of the ω-rule. In Plotkin [1974], however, some impressive universal generators were constructed for which rule (1) in fact does not hold.

This ω-incompleteness had some repercussions on the notion of model of the λ-calculus. There are λ-models and λ-algebras. The λ-models are well-behaved and include Scott's set-theoretic models and the open term models. The λ-algebras are less well behaved and include closed term models. Nevertheless, the latter models have interesting properties, notably as pre-complete numerations in the sense of Ershov [1973/75/77]; see also Visser [1980]. In Barendregt [1977] and [1981] the notions of λ-model and λ-algebra are described in a correct but rather ad hoc syntactical manner. A nice description in first order logic of the notion of λ-model was given independently in Scott [1980] and Meyer [1982]. Koymans [1982] completed the story of finding the description of λ-calculus models. Based on work of Scott he gave a description of λ-algebras in terms of cartesian closed

[4] The construction was related to Kreisels HRO (hereditarily recursive operations), which forms a model of the typed λ-calculus, see Troelstra [1973].

categories (CCC's)[5], in which λ-models form a special case.[6] The details of all this can be found in Barendregt [1984].

Finally, based on the work of Koymans [1982], in Curien [1986] so-called categorical combinators are developed for the use of implementations of functional programming languages. A successful application of this is CAM (categorical abstract machine) and the compiler CAML, used for implementing the proof-checker/developer COQ (based on the calculus of constructions, Coquand and Huet [1988]).

It also should be mentioned that in the proof of the partial validity of rule (1) the hypothesis ($FZ = GZ$, for all closed terms Z) was used in an extremely weak form: only one special closed term Z was used. Kreisel insisted that I should try to make use of more arguments in order to prove the full rule (1). I did not succeed and by Plotkins construction we know why. Nevertheless Kreisel turned out to be right that one can make use of more arguments. In Plotkin [1974a] it is proved that rule (1) is valid, provided that only one of F, G is not a universal generator, see Barendregt [1984], §17.3, for the details. In this proof the hypothesis of (1) is used for two different Z. In this line the best result is due to Nakajima [1975] and Wadsworth [1976]. They showed, using infinitely many Z, that the (axiom corresponding to the) rule (1) is valid in Scott's models D_∞; see also Barendregt [1984], §19.2.

So history proved that the interest of Kreisel in a neglected area of logic was fully justified.

Unsolvability

Another topic discussed with Kreisel for my thesis was the interpretation of the term $\Omega = (\lambda x.xx)(\lambda x.xx)$ in the recursion theoretic model. (This Ω is a $\lambda\mathsf{I}$-term and for these the candidate model is correct.) In the notation of Rogers [1967] this interpretation is

$$[\![\Omega]\!] = \varphi_e(e),$$

where e is such that $\varphi_e(x) = \varphi_x(x)$ for all $x \in \mathbb{N}$. The question was whether $[\![\Omega]\!]$ is defined or not.[7] It turned out that the answer depends on the choice of coding used to construct the universal function $\varphi_x(y)$. If a natural condition concerning lengths of computation is assumed for φ, then $[\![\Omega]\!]$ will be undefined; on the other

[5] A λ-algebra is a *reflexive* object in a CCC, i.e. an object D such that for some arrows $F : D \to [D \to D], G : [D \to D] \to D$ one has $F \circ G = id_{[D\to D]}$.

[6] A λ-model is a reflexive object D *having enough points*, i.e.

$$\forall f, g : D \to D[[\forall z : 1 \to D\ f \circ z = g \circ z] \ \Rightarrow\ f = g],$$

where 1 is the terminal object. Notice the relation with rule (1).

[7] This is related to a problem of Henkin, asking in arithmetic the provability of a statement H such that $\vdash H \leftrightarrow \Box H$, where $\Box H$ denotes formalized provability of H. As shown by Löb [1955] this is always the case.

hand for some 'non-standard' choices of φ this assumption is not valid and $[\![\Omega]\!]$ can be an arbitrary natural number[8]. See Barendregt [1975] for details.

Based on this result it follows that the interpretation of all λ-terms without normal form in the recursion theoretic structure with natural φ is undefined. As a consequence it seems to follow that terms without a normal form can be equated consistently.[9] This, however, is not the case. For example the equation between terms without a normal form

$$\lambda x.x\Omega\mathsf{true} = \lambda x.x\Omega\mathsf{false}$$

immediately gives $\mathsf{true} = \mathsf{false}$.[10] Analyzing the situation one sees that the two terms are *solvable*: $(\lambda x.x\Omega\mathsf{true})\vec{P}$ has a normal form for some $\vec{P}$ (and similarly for the other one). This notion turned out to be fruitful. All unsolvable terms can be identified consistently.[11] The consistency of the identification of unsolvable terms was proved later in Hyland [1976] and in Wadsworth [1976] by semantic methods (because $[\![M]\!]^{D_\infty} = \bot$, for M unsolvable). Moreover in these two papers it is proved that for closed terms M, N one has

$$\begin{array}{rcl} D_\infty \models M = N & \Leftrightarrow & \forall F[FM \text{ is solvable } \Leftrightarrow\ FN \text{ is solvable}] \\ & \Leftrightarrow & M = N \text{ belongs to the unique maximally consistent} \\ & & \text{extension of the } \lambda\text{-calculus equating the unsolvables.} \end{array}$$

In this way solvability is a natural organizing principle for semantics of the λ-calculus. Along similar lines in Abramsky and Ong [1993] an alternative semantics is introduced that reflects features of implementations of lazy functional programming languages (one does not reduce 'under' a λ).

3. GK: person and influence

The person

Who is Georg Kreisel? Although the answer should be given by professional biographers, let me make some personal remarks.

Surely Kreisel is one of the most remarkable and enigmatic figures among logicians (and non-logicians). His behavior is non-conventional. Take for example

[8] This non-standard φ is nevertheless an acceptable enumeration of the partial recursive functions of one argument in the sense of Rogers [1967].

[9] This presupposes that the recursion theoretic interpretation is a model for the λK-calculus.

[10] One needs an application to the combinator $\mathsf{K}_* \equiv \lambda xy.y$, which is a λK-term. In fact the recursion theoretic model does work for the λI-calculus. Hence we have the consistency of the λI-calculus $+ \{M = N \mid M, N$ have no normal form$\}$.

[11] In order to prove this one needs the so-called 'genericity lemma': if M is unsolvable and N a normal form, then $FM = N \ \Rightarrow\ Fx = N$. In the λI-calculus the unsolvable terms are exactly the terms without normal forms, see Barendregt [1973]. This gives an alternative proof of the consistency of the theory mentioned in footnote 10.

his daily sleeping time: Kreisel goes to bed at 9:00 p.m. Sometimes I suspect that for him this is a convenient way to avoid social obligations and to get some extra work done.[12] In any case it is a fact, that Kreisel receives daily more than a dozen letters and/or articles. All this correspondence is usually answered by return mail.[13] This requires a discipline and concentration that I have experienced otherwise only among zen monks.

Another characteristic is that Kreisel seems to like to create a certain distance between himself and persons that he does meet. In many cases this is indeed the case. At the same time this creation of distance is applied to his own emotions as well. This quality I have experienced otherwise only in theravadin monks, albeit that the latter use a somewhat different method for doing so. Kreisels method to successfully keep a proper distance from his emotions is by a logical analysis. If this is skillfully done, then it is possible to disintegrate one's emotions into smithereens. And the resulting parts and pieces are harmless.[14]

Let me give some examples. Being in a certain country, Kreisel was asked by the late professor X—locally a well-known logician—the following. "Perhaps it is a stupid question, but can you tell me why this and this." Kreisels reaction: "Ah, this question is not stupid at all, not stupid at all. The matter is so and so." The questioner continued eagerly: "Hence one also has that and that?" At this point Kreisel remarked in a flash: "Now *that* is a stupid question!"

This is what I would call creating a distance. And in this case Kreisel did so justifiedly: one does not want to be close to someone that either asks stupid questions while he knows it or wants to show off how smart he is. Kreisel gave this professor X both what he had hoped to hear and what he had feared to hear.

An example of Kreisels way to decompose his own emotions is more difficult to give. Let me make it clear that his mastery of argumentation is of a high standard.[15] Given this skill, he can accomplish a lot.[16] A good example of an analysis of emotions into nothingness is the way Kreisel made the son of the baroness eat his pudding. But remember, the boy was only six. Given the sophistication of Kreisels arguments one can imagine what is needed for a convincing transcending analysis of his own emotions. It is beyond my capacities to reproduce any of the cases in which I have witnessed this remarkable auto-analysis of Kreisel.[17]

[12]This hypothesis, however, I have not been able to verify.

[13]In these answers Kreisel often asks questions about a paper that are difficult for the author to answer.

[14]The theravadin monks also analyze emotions into components, not by ratio but by *insight* based on *mindfulness.* As opposed to Kreisels analysis based on logic, this requires less energy but a longer practice.

[15]Kreisel obtained some of his education from Jesuits.

[16]Due to an academic disagreement a collegue of Kreisel almost challenged him to court. It would have been interesting to compare the arguing skills of professional lawyers with that of Kreisel.

[17]In order to get an approximate idea of this phenomenon, one should read *Der Mann ohne*

Influence

One of the main pieces of advice that Kreisel gave me, was to use *reflection*. He claimed that logicians are often too busy with technical details in order to take some distance from their work. This way they lose a chance of obtaining better results.

An example of this is the following. In Barendregt [1973] I proved[18] that in the λI-calculus a term M has a normal form iff M is solvable. As title of the paper I chose: *'A characterization of terms in the λI-calculus having a normal form'*. Kreisel thought this was a particularly bad title. With a little more thought I could have given a much more significant and memorable one. Indeed, from the well-known result of Böhm [1967] and my own result that unsolvable terms can be consistently equated it follows that the λI-calculus plus η-conversion plus the equation of terms without a normal form constitute a Hilbert-Post complete theory.[19] So the title should have been something like: *'A natural Hilbert-Post complete extension of the λI-calculus'*.

Kreisel often made me aware of my academic career. "Take for example Heyting", he once said, "with results not more technical than yours he became well-known." Also Kreisel emphasized that one should publish one's results as soon as possible. With the present 'publish or perish' insanity,[20] this may sound obvious, but in 1971 it was not.

Each time I would see Kreisel again, after a few days, a few months or a few years, he would ask me "What is new?", cutting through our natural tendency to remain with our attachments. In fact, both scientifically and personally he always emphasizes *change*.

Claiming that all the advice that Kreisel had given me would fall under the heading 'Influence' is too much to be said. I wish it were true. But enough of his remarks are there in my memory, that occasionally things go along lines of his advice. Enough to have considerably profited from them.

Coda

Let me end with describing two sides of Kreisel that are apparently contradictory.

When I did drive with Kreisel in his car, I noticed that he would arrange things in a certain way, in order to have a better view. Remembering this a few years later when Kreisel came to Europe, I arranged things the same way in my car. His reaction: "Thank you very much. My cleaning girls never do this. Does one really need a Ph.D. in logic in order to do so?" This event, trivial as it may be, shows him as a pleasant companion.

Eigenschaften by R. Musil (Rowolt Verlag, 1952).

[18] Using induction over a Σ^0_4-predicate.

[19] I.e. a maximally consistent theory.

[20] Presently one publishes too much in proceedings; this is notably the case in computer science. Good ideas deserve to be published in journals.

One of the things that made him most angry, was my use of the sentence: "People will believe your opinion on this, because of your authority." At the time his strong reaction was not clear to me. This seems to show that he is a 'difficult' person and *not* a pleasant companion.[21]

The explanation of the apparent contradiction is easy. His negative feelings are caused by a general dislike of his. Kreisel abhors insincerity.[22] Since by and large most things in this world are done with insincere intentions, Kreisel is often 'difficult'. On the other hand Kreisel behaves well with the upper class, if they are sincerely upper class, and with the middle and lower classes, if they are sincerely so.[23]

The author Iris Murdoch wrote a novel in which a figure 'Julius' is said to be inspired by Kreisel. She wrote about this person: 'He is one of the people that opens your mail in your absence. But he tells you later that he did'.[24] If one has a taste for his style of sincerity and irony, then Kreisel is a very stimulating person to have around, both scientifically and personally. My friends that I had introduced to Kreisel all were charmed by him.[25]

Reflecting on what I just wrote, I search for a counterexample. Is it really true that his being difficult is always caused by insincerity of others? It almost sounds too good to be true for an ordinary mortal. In the case of Kreisel I could find no counterexample. So at least it is consistent.

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[21]Once he said: "Because they do this, I will be difficult. And I can be difficult *as a woman*."

[22]Indeed, his anger was justified: one should not believe in someone because of authority, but because of that person's arguments.

[23]Kreisel gave genuine attention and protection to a secretary, that had problems with a boyfriend, and to a terminally ill student.

[24]*A fairly honorable defeat*, Penguin Books, 1972. Another of her books (*An accidental man*, Penguin Books, 1973) has as simple dedication 'To Kreisel'.

[25]E.g. (in this order) my father, the musician Nol Prager, my mother, the computer scientist Corrado Böhm, the conceptual artist Hans Koetsier. Buffee Nelson, then a three year old girl that later became my adopted daughter, once picked up the phone when Kreisel called. Before she passed the call to, me she happened to listen attentively. Later she asked: "Who was that?" When I had told her, she said: "I like Kreisel!"

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